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DISTRIBUTION OF FOUNDATION LOADS IN TERMS OF SOIL-STRUCTURE INTERACTION

By

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ABSTRACT

Rational solution for the problem of contact pressure determination, in accordance with the soil-structure interaction, is presented here. The structure is assumed to be hypothetically supported on two or three yielding points, to render it the requirements of a main system. The contact pressure is substituted by a group of concentrated loads, whose magnitudes are determined from verification of the statical equilibrium condition and the combined kinematic condition of the whole system. The method of solution is programmed, in Fortran language, for direct application on electronic computer. This programme is included. Also a practical example is given for demonestrating the method of application.

INTRODUCTION

When a structure is founded on a yielding subsoil, it would normally undergo unequal settlement. The inner parts of the structure are more likely to settle; or have a tendency to settle, more than the outer parts. If the structure is not totally flexible, a new contact pressure distribution would develop, to render both the

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structure foundation plane and the subgrade surface an identical common shape. Consequently in any realistic evaluation of the contact pressures we should take into consideration the interaction between soil and superimposed structure. This may be done by investigating the effect of soil deformations upon differential settlement and the induced stresses and strains in the structure and the resulting redistribution of structural loads transmitted to the soil, due to stiffness of structure. This approach would certainly yield more accurate design rather than that obtained by introducing simplified assumptions, such as uniform or other arbitrary soil contact pressures; or performing analyses based on elastic subgrade reaction criterion.

Computing contact pressure, stresses and strains developing in loaded bases, resting over semi-infinite elastic medium, has formed a challenging topic for mathematicians during the last forty years. Among the contributers to this problem are Browicka (1939), who derived the solution for contact pressure below rigid smooth strip base due to symmetrical vertical loading, and Muskhelishvill (1963) who solved the same case due to moment loading. The latter in his work (1963) obtained the solutions for symmetrical vertical loading and moment loading acting on rough rigid strip. These contributions were based on 2-dimensional analyses.

With regard to 3-dimensional problems with polar symmetry, the case of rigid elliptic base loaded by symmetrical vertical loading has been considered by Schiffman and Aggarwala (1961), while Muki (1961) obtained the distribution of normal contact pressure beneath rigid circular area. The problem of circular rigid ring has been solved by Egorov (1965), while the case of rigid rectangular area loaded by symmetrical vertical loading has been solved approximately by Whitman and Richand (1967).

STATEMENT OF PROBLEM

As shown before, the distribution of contact pressure, computed on the basis of unyielding foundation, would no longer hold, once the structure skeleton is excited by additional stresses and strains, due to the tendency of the subgrade strata to undergo deformed shape under the structure. The deviation of the actual contact pressure from the computed one is mainly dependant on two factors; namely the overall stiffness of the building i.e. its load-deformation relationship, and secondly the final deformed shape of foundation.

The first factor is merely a strength of material problem, and by means of modern techniques applied in stress analysis, such as Finite Element Methods; it can be determined even for the most complicated structural systems. The second factor; which is the foundation settlement, is a function of both the subsoil moduli, and the final load distribution exterted by the structure on the soil. The latter is actually the unknown contact pressure, needed to be determined.

Normally, for such problems, whose solution procedure necessitates the knowledge of the final result before hand, solution could be obtained by assuming approximate value for the required result and proceeding with the calculation till we arrive to the result. Then comparing the obtained result with the assumed one, if they do not coincide, we make a better estimation for the assumed value and repeat this processes several times till the comparison shows that a close result is achieved.

Application of this trial and error method to unsymmetrical 3-dimensional problems of contact pressure determination, is by all means unpractical. This is because computations included are extremely tedius and the number of trials is unlimited since unsuccessful trials can hardly give indications for better estimation of the exact pressure distribution.

Taking these complications into account, a completely new alternative method is suggested here, that solves the problem algebraically. The method is tested on some actual problems, and proved to be very efficient, and even applicable to cases that have not been solved before. However, the calculations involved normally requires the use of electronic computer.

STRUCTURE MAIN SYSTEM

Let us consider a generalized structure of any irregular shape and stiffness, like that shown in Figure $(1)_{a\&B}$. This structure rests on compressible soil strata, so it exerts on the soil, at foundation level, unknown contact pressure, which is required to be evaluated.

To simplify the problem from the statical point of view, the actual structure is considered to be hypothetically supported on an appropriate number of supports, that renders it externally determinate; Fig. 1-c. Also the structure is regarded to be acted on by two different groups of loading. The first is the P-load group, which represents the loading due to the building own weight and the externally applied loads and pressures on the superstructure, whose components are fully known in magnitude and point of action. The second is the F-load group, which stands for the soil reaction; i.e. contact pressure acting on foundation base. The components of the F-load group are assumed to be a number of concentrated loads; namely F_1, F_2, F_3.... etc... F_n, acting at points f_1, f_2, f_3...etc... f_n, respectively. Hence the foundation sole plane or planes is hypothetically divided to an "n" number of relatively small elementary areas, and the reaction stress acting on each area is substituted by its resultant load "F". The components F_1, F_2...F_n loads are unknown in value, but their points of action "f" may be assumed, at the time being, acting at the centroid of the elementary areas.

Again, the points of action of the hypothetical supports are chosen to coincide with any of the points of action of the "F" load system, say f_1, f_{I3} and/or f_{I2}, i.e. the reactions of the hypothetical supports are considered belonging to the F-loading system components.

Like in any statically indeterminate structure, we can theoretically convert the redundant reactions of the "F" load group to forces, unknown in value. Thus the number of "F" loads in excess of 2, in a two dimensional problems, or 3, in three dimensional problems, has to be regarded as if they are unknown loads acting upwards. Consequently, the structure under consideration, would be converted to a main system, which is externally a statically determinate one.

5- GOVERNING EQUATIONS

5.1- ABSOLUTE SETTLEMENT EQUATIONS:

To start with, we shall deal with the effect of a single normal concentrated load and uniform distributed load on a semi-infinite mass of soil medium.

The particular problem of a concentrated load has been dealt with by Boussinesq (see e.g. Poulos H.G. & Davis, E.H., 1974) who obtained rigorously closed mathematical expressions for the stresses and strains at any point within the semi-infinite continuous mass. This work was extended afterwards by other workers, to cover many cases of loading and deflections. What interests us in these results is the vertical deflection of a general point on the top surface of semi-infinite mass.

From Boussinesq work, it follows that:

$$S = \frac{1 - u^2}{\pi . E} \times \frac{F}{a} \qquad \ldots (1)$$

where

- S denotes vertical displacement; i.e. settlement; of the subgrade surface,
- E denotes Modulus of elasticity of subgrade medium,

u denotes Poisson's ratio of subgrade medium,

F denotes applied load or force,

a denotes distance between the point of action of load (F) and the point at which settlement (S) is calculated.

Also, Tsytovich (1963) has worked out the required vertical displacement for certain distributed load areas. For the case of points at center and corner of rectangular areas & at center and edge of circular areas, that are uniformly loaded, he gave the following formula; making minor changes in the notation of the text:

$$S = \frac{2\,dp}{E}(1 - u^2)\,K_o \qquad \ldots\ldots (2)$$

Where

d denotes smaller side length of loaded rectangular area,

p denotes intensity of uniform distributed load,

K_o denotes a dimensionless influence factor computed for a range of areas. (table 2-2 page 86, Harr,1966).

The dimensionless factor K_o, in equation(2), is given in the original text of Tsytovich (1963) in a tabular form, in terms of the aspect ratio of the rectangular load area (r); (r = b/d). Here K_o has been substituted by a mathematical fraction obtained by regression technique, and that fits the tabular values very accurately.

The substitution of K_o and the pressure intensity (p) in Eq. (2) yields:

$$S = \frac{F}{b} \cdot \frac{1}{E}(1-u^2) \cdot (1.1142 + 0.6035 \times \ln \frac{b}{d}) \ldots \quad (3)$$

Where

b denotes longer side length of loaded rectangular area,

ln denotes Naperian (natural) logarithm.

By applying the rule of superposition; equations (1) & (3) can express the absolute vertical settlement 'S_i', at any point (f_i) at foundation level due to the F - system of loading, as follows:

$$S_i = \frac{1-u^2}{E}\left[\frac{F_1}{\pi\, a_{i,1}} + \frac{F_2}{\pi \cdot a_{i,2}} \ldots + \frac{F_i}{b_i} \cdot (1.1142 + 0.6035 \times \ln \frac{b_i}{d_i}) + \ldots \frac{F_n}{\pi \cdot a_{i,n}}\right] \quad \ldots .(4)$$

where

S_i	denotes absolute vertical settlement at point (f_i) due to components of the F-load system.
$a_{i,1}, a_{i,2} \ldots a_{i,n}$	denote distance between point (f_i) and points $(f_1), (f_2) \ldots (f_n)$.
b_i & d_i	denote longer & shorter side length of rectangular elementary area number (i), respectively.

It should be noted that equation (1) could be used always in computing the settlements resulting from the F-load components, except in case when the load element is acting directly over the point under consideration. Here the approximation involved in substituting the elementary area pressure by a concentrated load would be untolerable, and modified formula of Tsytovich (1963); equation (3); for distributed loading should be applied.

By similarity to equation (4), the following equations can be derived

$$\left.\begin{aligned} S_1 &= B_1 \cdot F_1 + C/a_{1,2} \cdot F_2 \ldots \text{ etc.} \ldots + C/a_{1,n} \cdot F_n \\ S_2 &= C/a_{2,1} \cdot F_1 + B_2 \cdot F_2 \ldots \text{ etc.} \ldots + C/a_{2,n} \cdot F_n \\ &\text{-----------------------------------} \\ &\text{-----------------------------------} \\ S_n &= C/a_{n,1} \cdot F_1 + C/a_{n,2} \cdot F_2 \ldots \text{etc.} \ldots + B_n \cdot F_n \end{aligned}\right\} \quad \ldots (5)$$

where

C denotes $(1 - u^2) / (\pi . E)$

B_i denotes $(1/b_i).(1.1142+0.6035 x \ln \frac{b_i}{d_i}).\frac{1-u^2}{E}$

a denotes distance as given in equation (4).

Again, equation (5) can be expressed in a matrix equation form, as follows

$$\begin{Bmatrix} S_1 \\ S_2 \\ -- \\ -- \\ -- \\ S_n \end{Bmatrix} = \begin{bmatrix} B_1 & C/a_{1,2} & .. \text{ etc.. } C/a_{1,n} \\ C/a_{2,1} & B_2 & ... \text{ etc.. } C/a_{2,n} \\ ------ & --------- & ---------- \\ ------ & --------- & ---------- \\ ------ & --------- & ---------- \\ C/a_{n,1} & C/a_{n,2} ... & \text{etc... } B_n \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ -- \\ -- \\ -- \\ F_n \end{Bmatrix} \quad (6)$$

Matrix Equation (6) expresses the absolute settlement equations for points $f_1, f_2 ... f_n$; under the F-load system. In other words, it gives values of settlement referred to the original surface of the unloaded semi-infinite medium. For sake of abbreviation, equation (6) is designated by

$$\{S\} = \left[S_{i,j}\right] \{F\} \quad (6)_a$$

It can be shown that $\left[S_{i,j}\right]$ is a symmetrical matrix, and its elements are the values of absolute settlement at points $f_1, f_2 ... f_n$, due to a load equal unity, applied once at a time at these points.

For example the first column and the first row in $\left[S_{i,j}\right]$ each is composed of settlement values at points f_1, f_2... f_n, respectively, due to unit load applied at point f_1, while the second column and the second row each is built from the settlements at the same points due to unit load acting at point f_2, and likewise. This is because of the validity of Maxwell theory for reciprocal deflections. It is clear that $\left[S_{i,j}\right]$ is independent on {F} values and its elements can be directly evaluated from the geometry of the foundation base and the values of elastic moduli of foundation medium.

5.2 REDUCED SETTLEMENT EQUATIONS:

In this context, by the term "reduced settlement" it is meant the induced settlement measured from plane, or from line, passing through the support points, after load application, as shown in Fig. 2.

Hence, it follows that

$$R_i = S_i - \bar{S}_i \qquad \ldots \qquad (7)$$

where

R_i denotes reduced vertical settlement at point (f_i)

$\bar{S}_i$ denotes absolute vertical settlement of plane/or line of supports at projection point of (f_i).

This means that the equations of reduced settlement of points f_1, f_2 ... f_n can be derived from the equations of absolute settlement; i.e. matrix equation (6); by subtracting the corresponding $\bar{S}_i$ values from elements of matrix $\left[S_{i,j}\right]$. Again, the reduced settlement equations could be summed up in matrix equation of similar form to equation (6), say

$$\{R\} = \left[R_{i,j}\right] \{F\} \qquad .... \qquad (8)$$

The elements of $\left[R_{i,j}\right]$ would be computed as given hereafter, one column at a time; taking the column No (i), in a 3-dimensional problem, as an example, for general application.

Here, a fixed cartesian coordinate system needs to be introduced, to deal with the problem in a systematic way. Assuming point (f_1), for example, is the origin of the system, and any two perpendicular directions on the foundation horizontal plane, to be the x- and y-directions. The direction of the z-axis, which is normal to the foundation base, would be devoted to the vertical settlement or deflection displacement. Then, from the geometry of the foundation, the coordinates of the points f_1, f_2 ... f_n would be computed; say x_1, x_2 ... x_n are their abscissas and y_1, y_2 ... y_n are their ordinates; respectively.

Since it is required to compute the elements of column No (i) in matrix $\left[R_{i,j}\right]$, a unit load is assumed applied at "f_i", and by means of equations (3) and/or (1), the corresponding absolute settlement at the hypothetical support points; f_1, f_{I2}, f_{I3}, due to this unit load, are calculated; say $S_{1,i}$, $S_{I2,i}$, $S_{I3,i}$; respectively. Consequently, the coordinates of these points would be $(0, 0, S_{1,i})$, $(x_{I2}, y_{I2}, S_{I2,i})$ & $(x_{I3}, y_{I3}, S_{I3,i})$, respectively.

Hence the equation of the plane passing through the three support points, after being deflected due to application of unit load at "f_i", could be derived. This is done by evaluating the value of the constants A_1, A_2 & A_3 in the general equation of plane in space, which reads

$$A_1X + A_2Y + A_3Z = 1 \quad \ldots\ldots \quad (9)$$

By substituting the coordinates of the support points, once at a time, in equation (9), we obtain the three simultaneous equations which are given here-after:

$$\left.\begin{array}{llll} 0 & + \quad 0 & + A_3.S_{1,i} & = 1 \\ A_1.x_{I2} & + A_2.y_{I2} & + A_3.S_{I2,i} & = 1 \\ A_2.x_{I3} & + A_2.y_{I3} & + A_3.S_{I3,i} & = 1 \end{array}\right\} \ldots \quad (10)$$

Solution of equations (10), yields the values of constants of equation 13; i.e. A_1, A_2 & A_3.

Then; by substituting with X & Y-coordinates of points f_1, f_2 ... f_n, once at a time, in the equation of plane of supports No (9), the vertical settlement of plane of supports at the projection of f_1, f_2 ... f_n, could be evaluated; say $\bar{S}_{1,i}$, $\bar{S}_{2,i}$ ... $\bar{S}_{n,i}$, respectively. Referring to equation (6) and equation (7), it follows that the column No (i) in the matrix $\left[R_{i,j}\right]$ of equation (8) would then read,

$$\begin{Bmatrix} (C/a_{1,i} - \bar{S}_{1,i}) \\ (C/a_{2,i} - \bar{S}_{2,i}) \\ \text{-------------} \\ \text{etc.} \\ \text{-------------} \\ (B_i \quad - \bar{S}_{i,i}) \\ \text{-------------} \\ \text{etc.} \\ \text{-------------} \\ (C/a_{n,i} - \bar{S}_{n,i}) \end{Bmatrix}$$

Hence the matrix equation for reduced settlement would be

$$\begin{Bmatrix} R_1 \\ R_2 \\ -- \\ -- \\ -- \\ R_n \end{Bmatrix} = \begin{bmatrix} B_1 \quad -\bar{S}_{1,1} & C/a_{1,2}-\bar{S}_{1,2} & .etc. & C/a_{1,n}-S_{1,n} \\ C/a_{2,1}-\bar{S}_{2,1} & B_2 \quad -\bar{S}_{2,2} & .etc. & C/a_{2,n}-\bar{S}_{2,n} \\ ---- & ---- & --- & ---- \\ ---- & ---- & --- & ---- \\ ---- & ---- & --- & ---- \\ C/a_{n,1}-\bar{S}_{n,1} & C/a_{n,2}-\bar{S}_{n,2} & & B_n \quad -\bar{S}_{n,n} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ -- \\ -- \\ -- \\ F_n \end{Bmatrix} \qquad \ldots \quad (8)_a$$

According to relative settlement definition, all the elements of matrix $\left[R_{i,j}\right]$ in equation $(8)_a$, falling in the rows assigned to R_1, R_{I2} & R_{I3} would be equal to zero.

This means that the matrix equation $(8)_a$ represents, in fact, a number of linear equations equals (n-2) or (n-3), pending on whether the problem under consideration is 2- or 3-dimensional one. Hence $\left[R_{i,j}\right]$ would be a rectangular matrix of $(\bar{n} \times \overline{n-2})$ or $(n \times \overline{n-3})$ elements.

5.3 DEFLECTION EQUATIONS

As mentioned before, the main system is assumed to be acted by two different loading groups; the P-load group and the F-load group. The deflections caused by these two load groups would be treated separately.

The values of the vertical deflection of points f_1, f_2 ... etc ... f_n, in the main system (item 4), due to the P-load components could be directly computed from the statics of the main system under consideration, say DP_1, DP_2 ... etc ... DP_n, respectively. This array of values would be designated by {DP}.

On the other hand, to establish the equations of vertical deflection of main system points f_1, f_2 ... f_n due to the unknown components of F-load group, we have to make two assumptions, to apply rule of superposition to structure stresses and strains. The first is that structure material is elastic and the second is that stresses and strains, developed in the structure, are within the proportionality limit. These assumptions are reasonable and practically valid to most structures.

Now, if a unit load is applied on the main system at point (f_i), it would produce at points f_1, f_2 ... etc ... f_n certain vertical deflections, say $\delta_{1,i}$, $\delta_{2,i}$, ... etc ... $\delta_{n,i}$, respectively. Hence by superposition and following the same concept and notational system, the vertical deflection of points f_1, f_2 .. etc .. f_n, due to the F-load components, could be expressed as given by matrix equation (11).

$$\begin{Bmatrix} DF_1 \\ DF_2 \\ --- \\ --- \\ --- \\ DF_3 \end{Bmatrix} = \begin{bmatrix} \delta_{1,1} & \delta_{1,2} & \dots \text{etc} \dots & \delta_{1,n} \\ \delta_{2,1} & \delta_{2,2} & \dots \text{etc} \dots & \delta_{2,n} \\ ----- & ----- & --- & ----- \\ ----- & ----- & --- & ----- \\ ----- & ----- & --- & ----- \\ \delta_{n,1} & \delta_{n,2} & \dots \text{etc} \dots & \delta_{n,n} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ -- \\ -- \\ -- \\ F_n \end{Bmatrix} \quad \dots (11)$$

where

DF_i denotes vertical deflection at point (f_i) due to F-load group.

$\delta_{i,j}$ denotes vertical deflection at point (f_i) due to unit load applied at point (f_j)

Equation (11) is designated by

$$\{DF\} = \left[\delta_{i,j}\right] \{F\} \quad \ldots \quad (11)_a$$

Like in case of matrix equation $(8)_a$, the $\left[\delta_{i,j}\right]$ in equation (11), is a rectangular matrix of ($N \times \overline{N-2}$) or ($N \times \overline{N-3}$) elements, since the elements of rows that correspond to the support points are zeros.

6. METHOD OF SOLUTION:

A solution for our main problem would be achieved if we obtain a number of algebraic equations in an equal number of unknowns reactions, F_1, F_2 ... F_n. That is to obtain an "n" number of equations in terms of F-load components. This may be done by equating the deflection of the main system to the reduced settlement of the subgrade surface, at each point of action of the F-load components. This satisfies the kinematic condition of the problem since the foundation base and soil medium surface are actually in close contact and both share one plane, and hence undergo identical deformed surface. Hence the compatibility equations would read,

$$\{DP\} + \{DF\} = \{R\} \quad \ldots \quad (12)$$

by substitution from equations (8) and (11) in equation (12), then

$$\{DP\} + \left[\delta_{i,j}\right] \cdot \{F\} = \left[R_{i,j}\right] \cdot \{F\} \quad \ldots(13)$$

$$\{DP\} = \left[R_{i,j}\right] \cdot \{F\} - \left[\delta_{i,j}\right] \cdot \{F\} \quad \ldots(14)$$

$$\{DP\} = \left[R_{i,j} - \delta_{i,j}\right] \cdot \{F\} \quad \ldots(15)$$

The matrix equation (15), as explained before, represents a number of equations equals (n-number of supports). By adding the equations of statical condition of equilibrium, which read

$$\Sigma P + \Sigma F = 0 \quad .. (16)$$

$$\Sigma M_x = 0 \quad .. (17)$$

and in case of 3-dimensional problems

$$\Sigma M_y = 0 \quad .. (18)$$

where

M_x & M_y denote first moment of all load components along the y-and x-axis, respectively.

to matrix equation (15), we would have the required "n" equation, whose all elements are determinable, except the F-element array. By solving the simultaneous equations (15), (16), (17) and/or (18), the values of the unknown F-load components would be determined.

7. METHOD OF APPLICATION

The solution of actual contact pressure problems by means of this method is merely a straight forward exercise. However, the process of application involves extended computational operations, since it includes solution of group of simultaneous equations, whose number normally runs into tens. Consequently, to render the method practical value, a "Fortran" programme has been developed, to carry out all computations.

The programme (refer to Appendix I) is designed, with the intention to be of a general application, as far as possible. The programme, first calculates the elements of the reduced settlement equations (equation (8)), from the coordinates of the elements of F-load group, i.e. (f_1, f_2 ... f_n), and the order of the hypothetical support reactions in the F-load group, i.e. subscriptions I_3 and /or I_2, which are fed in, as input data.

With respect to programming the operation of calculating the {DP} array, as well as establishing the matrix equations $\{DF\} = \left[\delta_{i,j}\right] \{F\}$, it was not possible to be fitted in the programme, in general application form. This is because these two processes are dependant on the characteristics of the structure as well as on the applied loading system. So this part has to be programmed each time, for every specific case, according to the particular configuration of structure and loading system, under consideration. The example programme, given in appendix I, works out the deflection matrix equation for the case of elastic beam founded over elastic semi-infinite mass; refer to Fig. 3-a.

The programme also compiles the reduced settlement equations and the deflection equations and obtains the (n) algebraic equations that satisfy the elastic and the statical equilibrium conditions. Finally the programme solves the n-algebraic equations and yields the required values of the unknown reaction elements F_1, F_2 ... F_n.

By dividing each reaction element (F) by its corresponding elementary area, the average contact pressure on every one of these foundation division areas would be defined. Then the contact pressure distribution, under the whole foundation, could be easily traced. It would be the continuous curve or surface that verifies the obtained average pressure values over the foundation small devision areas.

Here, the problem could be considered to be solved completely. However, for sake of perfection, a final check has to be made to get sure that certain approximation made in the method procedure is tolerable. This approximation is made when we assume that the resultant of pressure over each elementary area is sufficiently close to its centroid. If the contact pressure distribution diagram has no sharp slopes, then the already obtained result would be more than adequate for practical application. Otherwise, another run for the computer programme is needed, taking the points $f_1, f_2 \ldots f_n$ at points of action of resultants of contact pressure over the elementary areas, as yielded from the first run of computer programme.

7-1 STUDIED EXAMPLE

The structure dealt with, as an example, in this work is a solid rectangular beam, whose cross section is 100 cm x 100 cm, and its length is 1000 cm. The beam is acted by a group of nife edge loads applied at constant spacing, refer to Fig. (3-a). The elastic modili of the beam and of the supporting medium are given in the following table (1).

TABLE (1)

	Poisson's ratio	Youngs's Modulus of Elasticity
Beam	0.3	2040 KN/cm^2
Supporting medium (case 1)	0.3	0.05 KN/cm^2
Supporting medium (case 2)	0.3	10 KN/cm^2
Supporting medium (case 3)	0.3	2000 KN/cm^2

The results obtained from the application of the example programme to these three cases are given here-after in table (2). Also the contact pressure distributions per m for these cases are shown in Fig. (3-b).

TABLE (2)

Reaction (in K.N)	Case 1 (E=0.05 KN/cm^2)	Case 2 (E=10 KN/cm^2)	Case 3 (E=2000 KN/cm^2)
F_1	269.3	240.3	202.6
F_2	193.8	188.0	195.7
F_3	183.4	188.7	201.2
F_4	177.9	190.7	200.4
F_5	175.6	192.3	200.1
F_6	175.6	192.3	200.1
F_7	177.9	190.7	200.4
F_8	183.4	188.7	201.2
F_9	193.8	188.0	195.7
F_{10}	269.3	240.3	202.6

Due to the assumption, made in this work, that the supporting medium is a perfect elastic continuum, zones of high contact pressure would be developed at the edges of foundation. As seen in case (1) & (2); (Fig. 3); the contact pressure at the beginning and at the end of the tested beam tends to infinity. Practically, such unrealistic high pressure could be rectified if a ceiling value for the allowed contact pressure is introduced to the assumed stress-strain relationship. The maximum pressure value should be estimated according to the foundation bearing capacity.

8. CONCLUSIONS

The foundation contact pressure is replaced by a group of concentrated forces. The structure is regared to be resting on appropriate statically determinate supports, and the redundant reactions are theoretically converted to forces of unknown magnitudes. A matrix equation relating the settlement of the subgrade with the foundation loads is derived. The relationship between the deflection of the structure and both the subjected loads and foundation forces is established in the form of a matrix equation. By equating the settlement of the subgrade and the deflection of the structure, the required foundation forces are obtained.

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APPENDIX I - COMPUTER PROGRAM

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      MASTER (SOIL)
C     REDISTRIBUTION OF FOUNDATION LOADS AS CONNEC-
C     TED WITH SOIL-STRUCTURE INTERACTION
      DIMENSION PO(20),A(20),X(20,20),Y(20,20),
      AS(20,20),RS(20,20)
      DIMENSION ASMIL(20,20),RSMIL(20,20)
      DIMENSION E(20),A1(20),A2(20),Z1(20,20)
      DIMENSION XO(20),YO(20),XF(20),YF(20),COF(20,4)
      DIMENSION DIFF(20,20),DIFO(20,20),S ALF(20),S
      BET(20),GGMIL(21,21)
      DIMENSION BB(21,21),PF(20)
      DIMENSION CC(21)
C     M DENOTES NUMBER OF LOADING ELEMENTS (PO).
C     N DENOTES NUMBER OF FOUNDATION REACTION ELE-
      MENTS (PF).
C     U DENOTES MEUE OF SOIL.
C     E(IE) DENOTES LIST OF VALUES ASSIGNED TO MOD-
C     ULUS OF ELASTICITY OF SOIL.
C     L DENOTES NUMBER OF VALUES TO BE ASSIGNED TO
C     MODULUS OF ELASTICITY OF SOIL.
      READ(1,10)M,N,U
      READ(1,5)L
      READ(1,6)(E(IE),IE=1,L)
C     PO ARRAY DENOTES VALUES OF LOADING ELEMENTS.
      READ(1,15)(PO(I),I=1,M)
C     PF ARRAY DENOTES REQUIRED VALUES OF SUBGRADE
C     REACTION ACTING ON ELEMENTAL SOLE AREAS.
C     A1,A2 ARRAYS DENOTE THE SIDE LENGTHS,IN METER,
C     OF THE RECTANGULAR ELEMENTAL UNITS OR EQUIVA-
C     LENT RECTANGLES RESULTED FROM DIVIDING THE
C     FOUNDATION SOLE PLANE TO(N) DIVISIONS,A1 IS
C     BIGGER THAN A2.
```

```
      READ(1,20)(A1(I),I=1,N),(A2(I),I=1,N)
C     XO,YO ARRAYS DENOTE COORDINATES OF ELEMENTS
C     OF ARRAY(PO),W.R.T. ORIGIN COINCIDING WITH
C     POINT OF ACTION OF PF(1).
      READ(1,25)(XO(I),I=1,M),(YO(I),I=1,M)
C     XF & YF ARRAYS DENOTE COORDINATES OF ELE-
      MENTS OF ARRAY(PF),W.R.T. ORIGIN COINCIDING
      WITH POINT OF ACTION OF PF(1).
      READ(1,30)(XF(I),I=1,N),(YF(I),I=1,N)
C     CONSTRUCTION OF X,Y MATRICES,THAT IS TO SAY
C     MATRICES OF XF & YF COORDINATES W.R.T.
C     VARIOUS ORIGINS LOCATED AT POINT OF ACTION
C     OF PF(1),PF(2),...,PF(N),ONE AT A TIME-
C     (XF(NO. OF POINT USED AS ORIGIN,NO. OF PF
C     WHOSE XF IS CONSIDERED)).
      DO 60 I=1,N
      X(1,I)=XF(I)
   60 Y(1,I)=YF(I)
      DO 355 I=2,N
      DO 355 J=1,N
      X(I,J)=X(1,J)-X(1,I)
      Y(I,J)=Y(1,J)-Y(1,I)
  355 CONTINUE
C     (AS) MATRIX DENOTES VALUES OF ABSOLUTE
      SETTLEMENT.
C     CALCULATING THE ELEMENTS OF (AS) MATRIX.
C     THE NUMBER OF THE SELECTED THREE REFERENCE
C     POINTS ARE,NO. 1, NO. I2 AND NO. I3,AT
C     WHICH PE(1),PF(I2)&PF(I3) ARE ACTING.
      READ(1,45)I2,I3
      IE=0
 1000 IE-IE+1
      EM1=(1U**2)/E(IE)
      EM2=EM1/3.14159
      DO 555 I=1,N
      DO 555 J=1,N
```

```
      IF(J.NE.I)GO TO 444
      RA=A1(I)/A2(I)
      KO=1.1142+0.6095*ALOG(RA)
      AS(I,I)=EM1*KO/A1(I)
      ASMIL(I,I)=AS(I,I)*1000
      GO TO 555
  444 R=(X(I,J)**2+Y(I,J)**2)**0.5
      AS(I,J)=EM2/R
      ASMIL(I,J)=AS(I,J)*1000
  555 CONTINUE
      WRITE(2,35)
      WRITE(2,40)((ASMIL(I,J),J=1,N),I=1,N)
C     RS MATRIX DENOTES VALUES OF REDUCES
C     SETTLEMENT OF SUBGRADE DUE TO UNIT LOAD
C     APPLIED AT VARIOUS (PF)-POINTS OF ACTION.
C     CALCULATING THE ELEMENTS OF (RS) MATRIX.
C     ASSUME EQUATION OF PLANE PASSING THROUGH
C     REFERENCE POINTS AFTER SETTLEMENT IS
C     A1 X + B1 Y + C1 Z = 1,WITH ORIGIN AT
C     POINT OF APPLICATION OF PF(1).
C     BB(1),BB(2),BB(3) DENOTE A1,B1,C1
C     RESPECTIVELY.
      DO 710 J=1,N
      BB(3)=1.0/AS(1,J)
      IF(I2.EQ.I3)GO TO 230
      BB(2)=(1-BB(3)*AS(I2,J)*X(1,I3)-
     +(1-BB(3)*AS(I3,J)*X(1,I2))/
     + (Y(1,I2)*X(1,I3)-Y(1,I3)*X(1,I2))
      BB(1)=(1-BB(3)*AS(I3,J)-BB(2)*Y(1,I3))/
     +X(1,I3)
      GO TO 240
  230 BB(1)=(1-BB(3)*AS(I2,J))/X(1,I2)
      BB(2)=0.0
C     Z1 DENOTES AS-RS
  240 WRITE(2,50)(BB(I),I=1,3)
      DO 710 I=1,N
      Z1(I,J)=(1-BB(1)*X(1,I)-BB(2)*Y(1,I))/BB(3)
      RS(I,J)=AS(I,J)-Z1(I,J)
      RSMIL(I,J)=RS(I,J)*1000
```

```
  710 CONTINUE
      WRITE(2,75)
      WRITE(2,40)((RSMIL(I,J),J=1,N),I=1,N)
C     SUB-PROGRAM TO EVALUATE DIFLECTION
C     MATRICES AT POINTS OF ACTION OF F-LOADING
C     ELEMENTS FOR A LOADED BEAM FOUNDATION DUE
C     TO PF & PO SYSTEMS,NAMELY DIFF(I,J)&DIFO
C     (I,J).
      EME=1.7*100000.0*100000.0
C     EME DENOTES MOMENT OF INERTIA OF THE
C     BEAM*MOD OF ELASTICITY OF STRUC.MATERIAL,
C     IN(KN.CM**2), (1M*M CONC. BEAM).
      S1=XF(I2)
      DO 1919 I=1,N
      DO 1919 J=1,N
      S ALF(J)=S1-XF(J)
      S BET(I)=XF(I)
      SK=S ALF(J)+S BET(I)
      IF(SK.GT.S1)GO TO 1918
 1921 DIF F(I,J)=S ALF(J)*S BET(I)*(S1**2-S
     +ALF(J)**2-S BET(I)** 2)/
     +(6*S1*EME)
      GO TO 1919
 1918 S ALF(J)=XF(J)
      S BET(I)=S1-XF(I)
      GO TO 1921
 1919 CONTINUE
      WRITE(2,92)
      WRITE(2,94)((DIFF(I,J),J=1,N),I=1,N)
      DO 2919 I=1,N
      DO 2919 J=1,M
      S ALF(J)=S1-XO(J)
      S BET(I)=XF(I)
      SK=S ALF(J)+S BET(I)
      IF(SK.GT.S1)GO TO 2918
 2921 DIF O(I,J)=S ALF(J)*S BET(I)*(S1**2-S
     +ALF(J)**2-S BET(I)**2)/(6*S1
     +*EME)
      GO TO 2919
```

```
2918 S ALF(J)=XO(J)
     S BET(I)=S1-XF(I)
     GO TO 2921
2919 CONTINUE
     WRITE(2,96)
     WRITE(2,94)(DIFO(I,J),J=1,M),I=1,N)
     DO 830 I=1,N
     DO 830 J=1,N
     IF(I.EQ.1)GG(1,J)=1,0/1000.0
     IF(I.EQ.I2)GG(I2,J)=XF(J)/1000.0
     IF(I.EQ.I3.AND.I2.NE.I3)GG(I3,J)=
    +YF(J)/1000.0
     IF(I.NE.1.AND.I.NE.I2.AND.I.NE.I3)GG(I,J)=
    +RS(I,J)+DIF F(I,J)
 830 CONTINUE
     DO 840 I=1,N
     SUM=0.0
     DO855 J=1,M
     IF(I.EQ.1)SUM=SUM+PO(J)/1000.0
     IF(I.EQ.I2)SUM=SUM+PO(J)*XO(J)/1000.0
     IF(I.EQ.I3.AND.I2.NE.I3)SUM=SUM+
    +PO(J)*YO(J)/1000.0
     IF(I.NE.1.AND.I.NE.I2.AND.I.NE.I3)SUM=SUM+
    +PO(J)*DIF O(I,J)
 855 CONTINUE
     GG(I,N+1)=-SUM
 840 CONTINUE
     I = -50
     DO  7500    J=1,N+1
       GG (1,J) =  GG (1,J)  /  10.
      GG  (10,J) =    I      /( 1000.*1000.)
      I  =  I + 100
7500 CONTINUE
     GG (10,11)  =  -800.  / 1000.
     DO 3919 I=I,N
     DO 3919 J=1,N+1
3919 GGMIL(I,J)=GG(I,J)*1000.0
     WRITE(2,86)
     WRITE(2,88)((GGMIL(I,J),J=1,N+1)I=1,N)
     DO 7004 I=I,N
```

```
 7004 CC(I)=0.0
      KK11=1
      IF    (  KK11  .NE.  1  )  GO  TO  7003
      DO   618   I=1,N
  618 BB(I) = 200.
      GO   TO   616
 7003 CALL GAUSS (N,GG,BB)
  616 WRITE  (2,88)  (BB(I),I=1,N)
      DO 7005 I=1,N
 7005 CC(I)=CC(I)+BB(I)
      DO 7000 I=1,N
      SUM=0.0
      DO 7001 J=1,N
 7001 SUM=SUM+GG(I,J)*BB(J)
      GG(I,N+1)=GG(I,N+1)+SUM
 7000 CONTINUE
      KK11=KK11+1
      IF  (KK11.NE.21)  GO  TO  7003
      WRITE(2,88)(CC(I),I=1,N)
      IF(IE.NE.L) GO TO 1000
C
    5 FORMAT(1I2)
    6 FORMAT(10F8.2)
   10 FORMAT(2I3,2F8.4)
   15 FORMAT(10F8.3)
   20 FORMAT(10F8.3)
   25 FORMAT(10F8.3)
   30 FORMAT(10F8.3)
   35 FORMAT(///10X,'RESULTS'//10X,'(AS) VALUES
     +MULTIPLIED BY (1000 )')
   40 FORMAT(10X,10F8.4,/)
   45 FORMAT(2I2)
   50 FORMAT(///20X,'THE EQUATION OF THE PLANE
     +IS ',2X,E14.8,' X + ',
     +E14.8,' Y + ',2X,E14.8,' Z + - 1.0 = 0.0 ')
   72 FORMAT(/20X,F8.3)
   75 FORMAT (///10X,'(RS) VALUES MULTIPLIED BY
     +(1000 )',///)
   80 FORMAT(//10X,'THE ELEMENTS OF G MATRIX'//)
   86 FORMAT(//10X,'(GGMIL) VALUES MULTIPLIED BY
     +(1000 )',//)
   88 FORMAT(5X,10(F8.4,3X),3X,F9.4,/)
   92 FORMAT(//10X,'(DIF F) VQLUES ',//)
```

```
   94 FORMAT(5X,10(E12.6,2X),/)
   96 FORMAT(//10X,'(DIF O) VALUES ',//)
      STOP
      END

      SUBROUTINE GAUSS(MG,G,BB)
C     SOLVING SIMULTANEOUS EQUATIONS
      DIMENSION AA(100),BB(21),W(20),G(21,21)
      DO 177 I=1,MG
      DO 76  L=1,MG
      KA=I+MG (L-1)
   76 AA(KA)=G(I,L)
      BB(I)=-G(I,MG+1)
  177 CONTINUE
      CALL FPMGESOL(MG,1,0.000001,AA(1),BB(1),
      W(1),DET,IRANK,NRR)
      RETURN
      END
```

APPENDIX II - REFERENCES

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APPENDIX III - NOTATIONS

A_1, A_2 & A_3	=	constants of plane equation; equation (9) in the text.
a	=	distance between the point of action of load and the point at which settlement is calculated.
$a_{i,j}$	=	distance between two general points f_i & f_j.
b_i	=	longer side length of rectangular area number (i).
B_i	=	$(1/b_i).(1.1142+0.6035 \ln \frac{b_i}{d_i}). \frac{1-u^2}{E}$
C	=	$(1-u^2)/(E.\pi)$
d_i	=	shorter side length of rectangular area number (i).
$DF_1, DF_2 \ldots DF_n$	=	vertical deflection of main system at points $f_1, f_2 \ldots f_n$; respectively; due to all F-Load components.
$DP_1, DP_2 \ldots DF_n$	=	vertical deflection of main system of points $f_1, f_2 \ldots f_n$; respectively; due to all P-load components.
E	=	Young's modulus of elasticity medium.
F	=	concentrated load acting at foundation sole; F = p x area.
$F_1, F_2 \ldots F_n$	=	components of soil reaction F-load group, acting at centroid of elementary areas number 1,2 ... n, respectively.

$f_1, f_2 \ldots f_n$ = points of action of Loads F_1, F_2 ... F_n respectively.

f_i, f_j = two general points of action of any two components of the F-load system.

f_1, f_{I2} & f_{I3} = points of hypothetical supports.

i, j = subscriptions referring to two general points under consideration.

K_o = dimensionless influence factor in Tsytovich work (1963); equation (2).

ln = Naperian (natural) logarithm.

n = number of components of F-load group.

P = an external or structure own weight load; super-structure load.

p = intensity of external load per unit area.

R_i = reduced vertical settlement of subgrade surface, at point (f_i), due to F-load components.

r = aspect ratio of sides of rectangular area = b/d

S = absolute vertical settlement of subgrade surface.

S_c = absolute vertical settlement of subgrade surface, at centre of circular loaded area.

S_i = absolute vertical settlement of subgrade surface, at point(f_i), due to F-load components.

$S_{i,j}$ = absolute vertical settlement of subgrade surface, at point (f_i) due to unit load applied at point (f_j).

$\bar{S}_i$ = absolute vertical settlement of plane or line of supports at projection point of (f_i), due to F-load components.

$\bar{S}_{i,j}$ = absolute vertical settlement of plane or line of supports at projection point of (f_i), due to unit load applied at point (f_j).

u = Poisson's ratio of elastic medium.

$\delta_{i,j}$ = vertical deflection of main system at point (f_i), due to unit load applied at point (f_j).

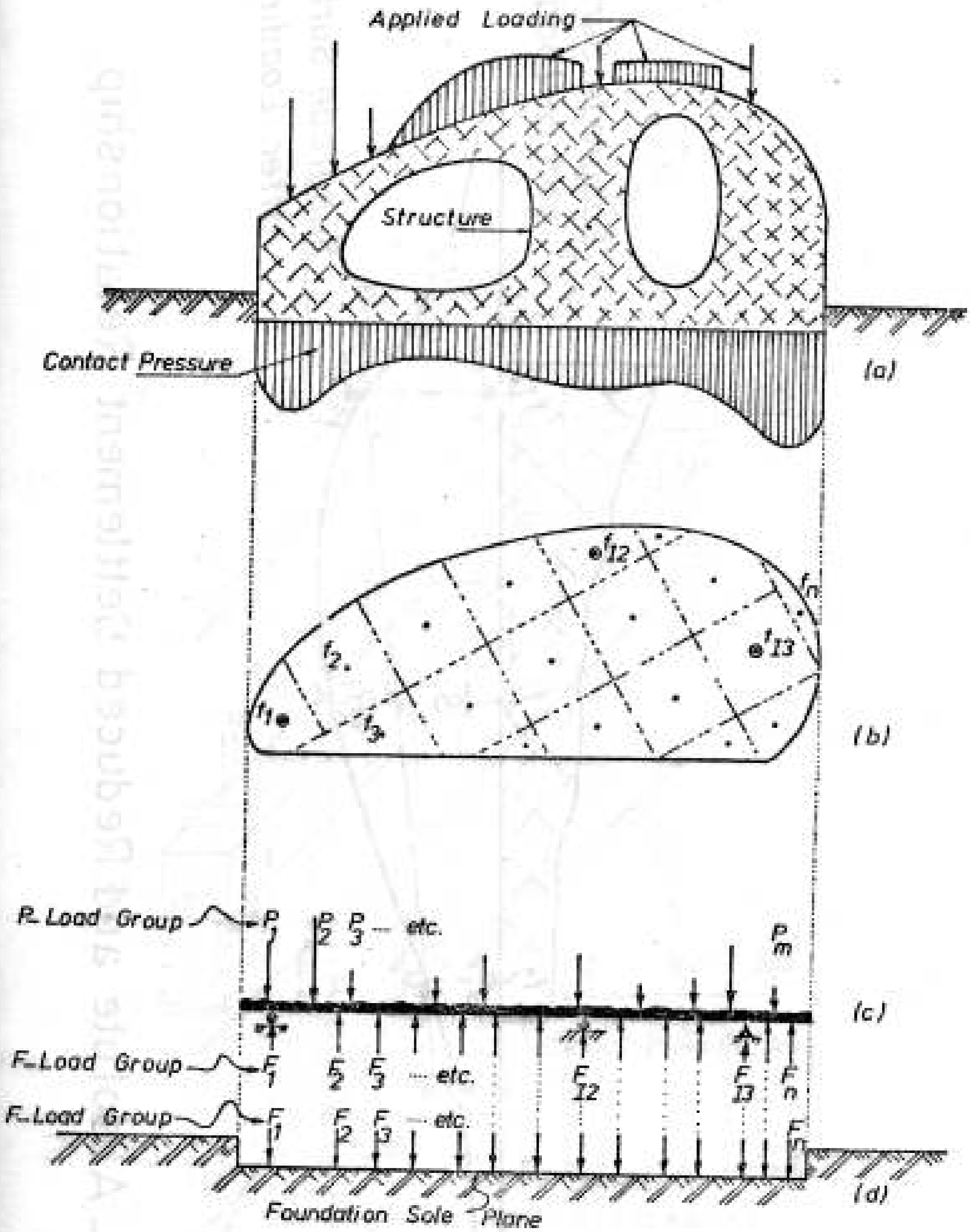

FIG.1. Structure of any Generalized Form.

(a) Elevation; (b) Plan; (c) Main System - Statically Determinate; (d) Loads Transmitted from Structure to Subgrade.

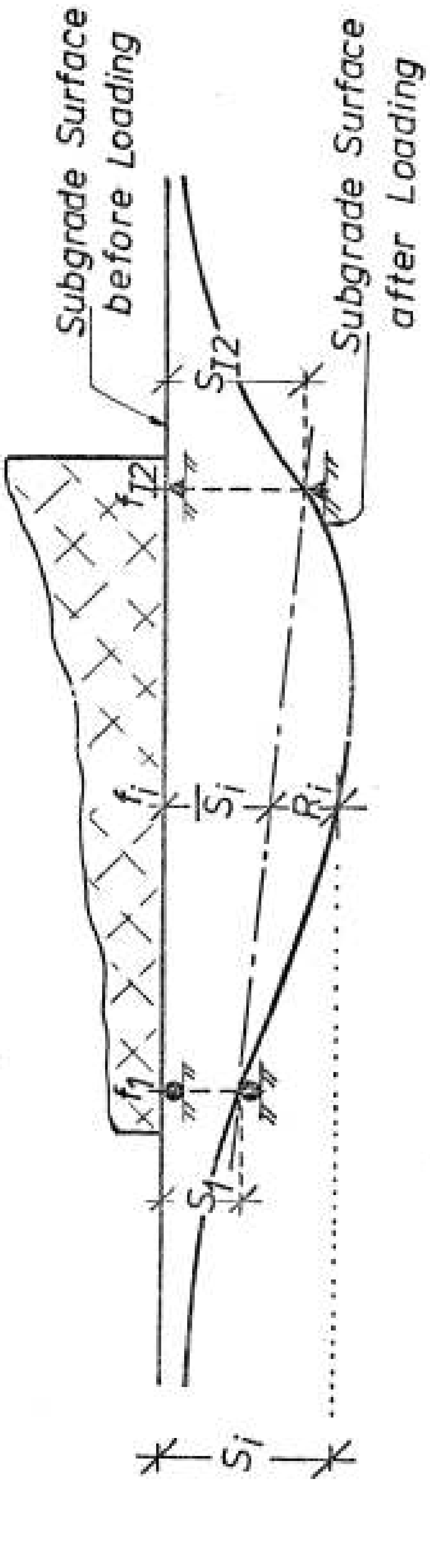

FIG. 2 . Absolute and Reduced Settlement Relationship.

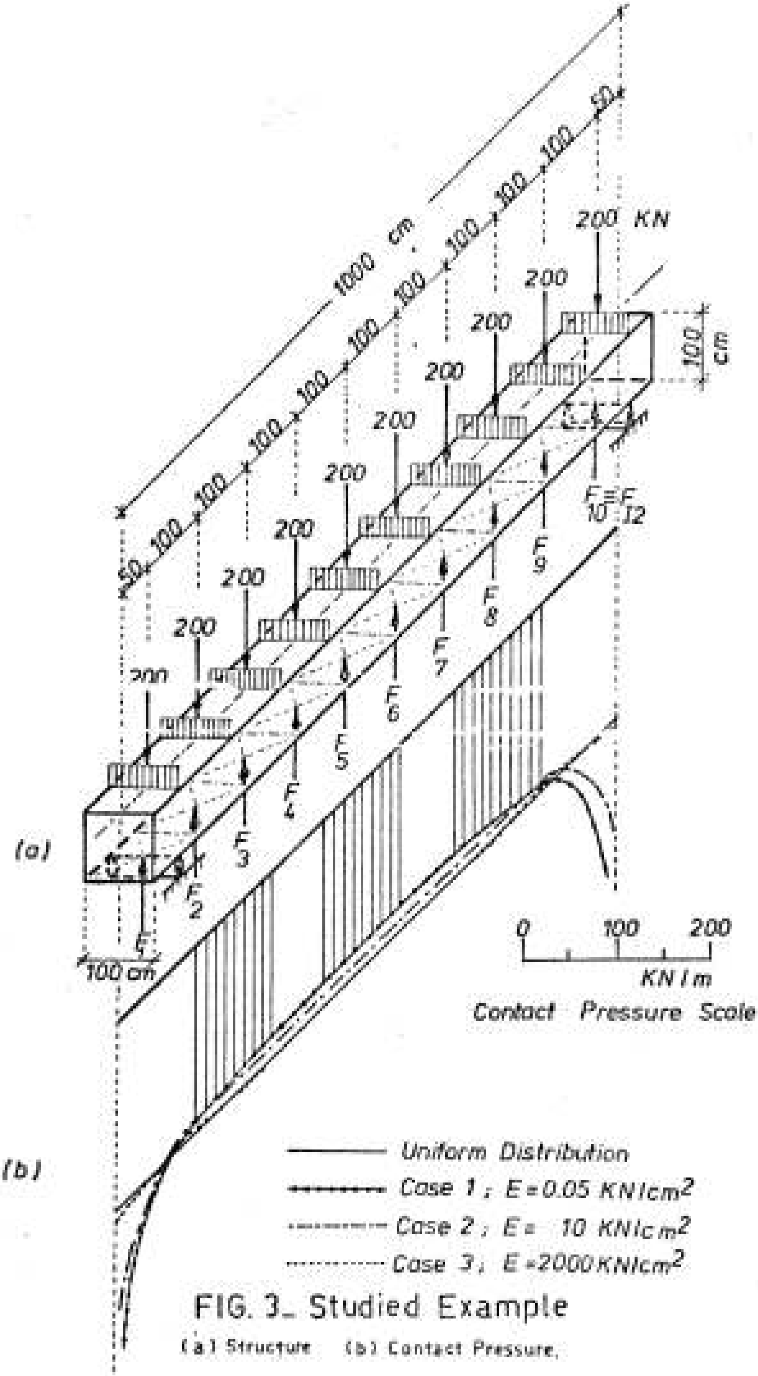

FIG. 3_ Studied Example

(a) Structure (b) Contact Pressure.