A RATIONAL METHOD FOR DETERMINING SOIL PRESSURE ON STRUCTURES

By
KHAFAGY; A.A.
Director of Coastal Research Institute,
Academy of Scientific Research & Technology,
Egypt.

Alexandria (1979)

ABSTRACT

A new technique for indirect measurement of intensity and distribution of lateral earth pre-ssure and contact pressures, acting on super-structure and foundation elements, is presented. The method concept originates from the virtual relationship between any loading system and the strain flow, developed throughout the loaded element.

The suggested method is applicable to any 2- or 3-dimensional elastic element of practical value. It is rather useful for testing laboratory physical models and prototype structures. The me-thod of application would normally require the use of electronic computer. A case of an assumed structure is investigated by the proposed method of analysis and given here as a demonestrating example.

INTRODUCTION

Needless to say how vital it is to find a way to determine accurately the soil induced pressures subjected to structures. During the last forty years tremendous efforts were devoted for this objective. Consequently, a good deal of progress was achieved, mostly in phases dealing with the role of the various factors pertinent to the induction of earth pressure in nature. Also, laboratory research work led to the development of big variety and various types of earth pressure gauges to carry out direct measurements. Unfortunately, the technique applied in these gauges has its limitations and drawbacks specially when used in connection with particulated continuum. Hence, earth pressure gauges are used now, rather with great caution, in research work only and can not be considered by any scale, adequate for general application.

In the curren. work, a method for deducing the actual earth pressure and loads acting on any structural member, from its state of excitation, is presented. Using the member stress or strain excitation is by no means a new approach. On the contrary, very early unsuccessful attempts adopted the same way of thinking and tried to compute the applied loading system from the member deformations, according to the well known differential equation given hereafter, which relates the applied loading system with the induced deflection, for two dimensional elements.

$$P = \frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2}\right) - \frac{d}{dx} \left(N \frac{dw}{dx}\right) \qquad \ldots \qquad (1)$$

where EI = flexural stiffness of loaded member.

w = transverse deflection of neutral axis.

N = axial tension along neutral axis

p = applied transverse load per unit length

x = the distance along the neutral axis

As the transverse deflection of neutral axis is not analytically revealable in practice, the loading diagram has to be acquired, according to equation "1", by graphical differentiation processes, performed on the approximately determined neutral axis curve. This procedure yielded always meaningless results, due to magnifying the excution errors during the graphical differentiation processes, that has to be carried out four successive times.

However, the problem is treated here from different scope. The applied loads and pressures are deduced from strain values, by a numerical analysis procedure. By tackling the solution in this way, excution errors due to graphical procedure are avoided. Also, the normal strain data used in the analysis, could be determined with high accuracy by means of modern strain measurement technique.

METHOD CONCEPT

Considering the case of a body of generalized form; fig "1", which is resting on either an appropriate supporting system that renders it externally statically determinate, or any other type of supporting system, whose supports are either totally rigid or elastically yielding ones that regain their original position by load relief. It is also assumed that the body itself has perfect elastic properties. This means that:

a. The rule of superposition for stress, strain and applied loading is valid; i.e. linear relationship between stress & strain or applied load & strain.

b. The body is capable to regain its original state and dimensions after load relief.

When the body is subjected to a single concentrated load "F_j", acting at a point "P_j"; refer to Fig. "1"; the whole body, in general, would undergo certain deformations. If location "L_i" defines a certain point on the body surface and a certain direction along which the state of strain is investigated, then "L_i" would experience, in general, certain normal strain variation say "$\xi_{i,j}$", which is compatible with the strain flow developed in the whole body. By the removal of load "F_j", the body would regain its original state and "$\bar{\xi}_{i,j}$" would also vanish. Whenever load "F_j" is applied again at "P_j" the location "L_i" would experience the same amount of strain variation "$\bar{\xi}_{i,j}$".

If load "F_j" is substituted by a unit load, i.e. a load whose scalar magnitude equals unity; acting at point "P_j" and in the same direction of "F_j", then due to the body's elastic properties, the location "L_i" would experience a normal strain variation "$\bar{\epsilon}_{i,j}$" and

$$\therefore \quad \epsilon_{i,j} = \bar{\epsilon}_{i,j} / f_j \qquad \ldots \quad (2)$$

where f_j = the scalar magnitude of load "F_j".

As shown above, if $\epsilon_{i,j}$ & $\bar{\epsilon}_{i,j}$ are known, the load scalar "f_j", for this single load case, could be computed from these strain values, by equation "2".

GOVERNING EQUATIONS

To study the case of elastic body of generalized shape when subjected to a group of "n" concentrated loads; say F_1, F_2 ... etc. ... F_n, acting at points P_1, P_2 ... etc. ... P_n, respectively; Fig. "2"; an "n" number of locations; say L_1, L_2 ... etc. ... L_n; on the body surface, should be selected for normal strain measurement. Due to the application of this group of loads, normal strain variation would, in general, take place at each of the locations L_1, L_2 ... L_n.

Because of the validity of the rule of superposition, the developed normal strain, resulted from the whole group of loads F_1, F_2 ... etc. F_n, at any location, would be equal to the algebraic sum of the normal strain induced by the individual loads F_1, F_2 ... and F_n, at thislocation, i.e.

$$\varepsilon_i = \sum_{j=1}^{j=n} \bar{\varepsilon}_{i.j} \qquad \ldots \quad (3)$$

where

$\bar{\varepsilon}_i$ = normal strain developed at location L_i, due to the whole load group F_1, F_2 ... F_n.

$\bar{\varepsilon}_{i,j}$ = normal strain developed at location L_i, due to load F_j.

i&j = digits that can be assigned to any number from "1" to "n".

$$\therefore \bar{\varepsilon}_i = \bar{\varepsilon}_{i,1} + \bar{\varepsilon}_{i,2} + .. + \bar{\varepsilon}_{i,j} + \ldots \bar{\varepsilon}_{i,n} \qquad (3)_a$$

But from equation "2" it follows that

$$\bar{\varepsilon}_{i,j} = f_j \cdot \varepsilon_{i,j} \qquad (2)_a$$

By applying the relationship given by equation $(2)_a$ to elements of equation $(3)_a$, we get

$$\bar{\varepsilon}_i = f_1 \cdot \varepsilon_{i,1} + f_2 \cdot \varepsilon_{i,2} + \ldots + f_n \cdot \varepsilon_{i,n} \qquad (4)_i$$

By assigning the digit (i) in equation $(4)_i$ to number 1,2 .... or n, we obtain the equations of the normal strain variation at location L_1, L_2 ... and L_n, as follows:

$$\bar{\varepsilon}_1 = f_1 \cdot \varepsilon_{1,1} + f_2 \cdot \varepsilon_{1,2} + \ldots + f_n \cdot \varepsilon_{1,n} \qquad (4)_1$$

$$\bar{\varepsilon}_2 = f_1 \cdot \varepsilon_{2,1} + f_2 \cdot \varepsilon_{2,2} + \ldots + f_n \cdot \varepsilon_{2,n} \qquad (4)_2$$

- -

- -

$$\& \quad \bar{\varepsilon}_n = f_1 \cdot \varepsilon_{n,1} + f_2 \cdot \varepsilon_{n,2} + \ldots + f_n \cdot \varepsilon_{n,n} \qquad (4)_n$$

The equations No. $(4)_1$ to $(4)_n$ forms a set of simultaneous algebraic equations that can be expressed in a matrix equation, as follows:

$$\begin{Bmatrix} \bar{\xi}_1 \\ \bar{\xi}_2 \\ -- \\ -- \\ \bar{\xi}_n \end{Bmatrix} = \begin{bmatrix} \xi_{1,1} & \xi_{1,2} & \cdots\cdots & \xi_{1,n} \\ \xi_{2,1} & \xi_{2,2} & \cdots\cdots & \xi_{2,n} \\ ----- & ---- & ------ & ---- \\ ----- & ---- & ------ & ---- \\ \xi_{n,1} & \xi_{n,2} & & \xi_{n,n} \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ -- \\ -- \\ f_n \end{Bmatrix} \quad (5)$$

Matrix equation (5) is the basic equation used in the present work, and although it has been derived from the case of concentrated loads action, its application would be generalized as shown later.

For sake of abbreviation, equation (5) would be designated by

$$\{\bar{\xi}\} = [\xi] \cdot \{f\} \quad (5)_a$$

The elements of the column matrix $\{\bar{\xi}\}$, in equation (5) are the values of the normal strain developed at L_1 to L_n due to application of the whole load system. These values are directly measurable from the loaded body.

The square matrix $[\xi]$, in equation (5), would be called the influence coefficient matrix. It is built up from the values of the strain developed at the various measuring locations "L_1 to L_n" due to the application of a unit load applied at the loading points "P_1 ... to P_n", one at a time in the sequence given above. The values of $[\xi]$ elements, could be either theoretically calculated by Statics and Strength of Materials or measured by an experimental calibration process for the body or a element under consideration, as discussed later, in some detail.

The column matrix $\{f\}$, in equation (5), is composed of the scalar magnitudes of the acting loads F_1 to F_n. If the values of the elements of both vector $\{\xi\}$ and square matrix $[\xi]$ are determined, equation (5) could be solved and elements of vector $\{f\}$ are then yielded.

GENERALIZATION OF THE LOADING SYSTEM

Equation (5) provides a mean to deduce the scalar magnitudes of the components of the subjected loading system from normal strain measurements, and is directly applicable to cases whose loading system is composed of concentrated loads of known direction and point of action. In case when the subjected loading system includes distributed loads, such as earth induced pressures, equation (5) may be still applied with adequate accuracy, if the distributed load is hypothetically converted to a group of concentrated loads.

To accomplish this, the area submitted to distributed loading would be hypothetically divided to a number of relatively small elementary areas. The smaller the areas the better the approximation would be. If the direction of pressure action is known, one concentrated load is to be assigned to each of these areas, otherwise 2 or 3 load components, parallel to fixed x, y and/or z axis, would be assigned to each area. The magnitudes of these concentrated loads are unknown, and would be equal to the resultants of pressure exerted on the elementary areas.

The equivalent point of action of these hypothetical concentrated loads are yet not known. To proceed with the analysis application, we assume, at the time being, that these loads are acting at the centroids of the corresponding elementary areas. This assumption is reasonably accurate for many cases. However it can be checked and modified, if necessary, after computing the load magnitudes and determining the distribution of the subjected pressure. If the obtained pressure distribution diagram has sharp or steep variations, that cause the points of action, of the resultant of pressure over the elementary areas, to be situated relatively away from the areas centroids, a repeating of the solution may be worthy, taking the point of action of loads as yielded from the first attempt.

By the above mentioned adaptation, Equation (5) becomes applicable to almost all types of loading systems.

CONSTRUCTION OF INFLUENCE COEFFICIENT MATRIX

The elements of the square matrix $[\xi]$, in equation (5), are evaluated whether theoretically or experimentally, by performing a number "n" of calibrating operations on the structure under consideration, or on a model. From each operation, the elements of one column of $[\xi]$ are computed. Taking, for example, the column number "i" in $[\xi]$; to find its elements, a unit load is applied at location "L_i". The strain values developed at locations, L_1, L_2, ... etc ... L_n; would be the required values $\xi_{1,i}$, $\xi_{2,i}$, ... etc. ... $\xi_{n,i}$; respectively. By similar way, the elements of the other columns could be determined.

SUPPORTING SYSTEM

The proceed with the calibration operations, the structure under consideration should be resting on a finite supporting system. In many cases, encountered in the field of geotechnique; like in case of sheet piles; the structure would be supported, somehow, by soil pressures that render it equilibrium.

To deal with these types of structures whose supporting systems are not definite, both the exerted loads and reactions are treated as unknown subjected forces. The structure would then be given any desired supporting system, that has to be statically determinate one, and the calibration process for evaluating the elements of $[\xi]$ is performed, either theoretically or experimentally. Then we proceed with the application of the method as normal.

The introduction of these unrealistic supports, during the process of construction of influence matrix, has no effect on the yielded results. This is because the structure under consideration would be actually in equilibrium, under the subjected loads and soil reactions, which are treated here as acting loads. So, the reactions of the introduced statically determinate hypothetical supports, for these loads, whose resultant is zero, would be also zero. Hence their absence during the process of measurement of $\{\xi\}$ values, would have no effect.

MATHEMATICAL CONSIDERATIONS

The effort for solving matrix equation (5) is depending on the order of $[\xi]$; i.e. $(n \times n = n^2)$. If 'n' is small; say 4 or less; equation "5" could be solved manually, but if it is of higher order, a digital computer might be required. Computers of moderate capacity could easily solve this type of equations even whose order "n^2" run in many hundreds.

However, an alternative equation for equation (5) is given hereafter, to minimize the use of computer, in case the method is used in experimental research programme that investigates a structure under series of cases of loadings with various load intensities.

From equation $(5)_a$, it follows that

$$\{f\} = [\xi]^{-1} \cdot \{\bar{\xi}\} \qquad \dots (6)$$

or

$$\{f\} = [\eta] \cdot \{\bar{\xi}\} \qquad \dots (6)_a$$

where

$[\eta] = [\xi]^{-1}$ = matrix which is the inverse of $[\xi]$.

If $[\eta]$ is determined, the required components of vector $\{f\}$ could be directly calculated by means of equation $(6)_a$, by simple multiplying operations.

From matrices algebra, the $[\eta]$ is also a square matrix of the same order as $[\xi]$; i.e. order (n x n).

Theoretically, elements of $[\eta]$ could be determined from the equation; (H.C. Martin, 1966);

$$[\eta] = \frac{\mathrm{adj}\,|\xi|}{|\xi|} \qquad \dots (7)$$

where

adj $|\xi|$ = the adjoint of $|\xi|$

$|\xi|$ = the determinant of the square array of $[\xi]$

Application of equation "7", for determination of $[\eta]$ is unpractical, since the evaluation of adj $|\xi|$ requires computing the values of a number of determinants equals n^2, each of order $(n-1)$.

Fortunately, the $[\xi]$ has special characteristics, that makes it possible to find its inverse $[\eta]$ by a much easier and practical procedure. Equation $(6)_a$ reads

$$\begin{Bmatrix} f_1 \\ f_2 \\ -- \\ f_n \end{Bmatrix} = \begin{bmatrix} \eta_{1,1} & \eta_{1,2} & \cdots & \eta_{1,n} \\ \eta_{2,1} & \eta_{2,2} & \cdots & \eta_{2,n} \\ ---- & ---- & ---- & ---- \\ \eta_{n,1} & \eta_{n,2} & \cdots & \eta_{n,n} \end{bmatrix} \begin{Bmatrix} \bar{\xi}_1 \\ \bar{\xi}_2 \\ -- \\ \bar{\xi}_n \end{Bmatrix} \quad \cdots (6)_b$$

where

$\eta_{i,j}$ = the element in $[\eta]$, whose row number equals (i), and column number equals (j).

Let us assume that, in the calibration process, a unit load is applied at "P_j". The developed strain in location; L_1, L_2 ... L_n; would be measured and are $\xi_{1,j}$, $\xi_{2,j}$ ... $\xi_{n,j}$, respectively.

By substituting with the values of applied load and measured strain values, which are known, in matrix equation $(6)_b$, we get:

$$\begin{Bmatrix} 0 \\ 0 \\ - \\ 1 \\ - \\ 0 \end{Bmatrix} = \begin{bmatrix} \eta_{1,1} & \eta_{1,2} & \cdots & \eta_{1,j} & \cdots & \eta_{1,n} \\ \eta_{2,1} & \eta_{2,2} & \cdots & \eta_{2,j} & \cdots & \eta_{2,n} \\ ---- & ---- & --- & ---- & --- & ---- \\ \eta_{j,1} & \eta_{j,2} & \cdots & \eta_{j,j} & \cdots & \eta_{j,n} \\ ---- & ---- & --- & ---- & --- & ---- \\ \eta_{n,1} & \eta_{n,2} & \cdots & \eta_{n,j} & \cdots & \eta_{n,n} \end{bmatrix} \begin{Bmatrix} \xi_{1,j} \\ \xi_{2,j} \\ ---- \\ \xi_{j,j} \\ ---- \\ \xi_{n,j} \end{Bmatrix} \quad (8)$$

Noting that the elements of matrix $[\eta]$, in matrix equation (8), are still unknown, while the vectors $\{f\}$ and $\{\xi\}$ are known. The equation of row No (j), of matrix equation "8", reads:

$$1 = \eta_{j,1} \cdot \xi_{1,j} + \eta_{j,2} \cdot \xi_{2,j} + \cdots + \eta_{j,j} \cdot \xi_{j,j} \cdots + \eta_{j,n} \cdot \xi_{n,j} \qquad (8)_a$$

Also, if the unit load acts at any point, other than No (j), say point No (i), then by substituting in matrix equation $(6)_b$, the equation of row No(j) would read:

$$0 = \eta_{j,1} \cdot \xi_{1.i} + \eta_{j,2} \cdot \xi_{2,i} + \cdots + \eta_{j,j} \cdot \xi_{j,i} \cdots + \eta_{j,n} \cdot \xi_{n,i} \qquad (8)_b$$

As (i) can take any value between "1" and "n", except (j), then the equation $(8)_a$ and equations given by $(8)_b$, can be written in the following matrix equation form

$$\begin{Bmatrix} 0 \\ 0 \\ - \\ - \\ 1 \\ - \\ 0 \end{Bmatrix} = \begin{bmatrix} \xi_{1,1} & \xi_{2,1} & \cdots & \xi_{j,1} & \cdots & \xi_{n,1} \\ \xi_{1,2} & \xi_{2,2} & \cdots & \xi_{j,2} & \cdots & \xi_{n,2} \\ ---- & ---- & --- & ---- & --- & ---- \\ ---- & ---- & --- & ---- & --- & ---- \\ \xi_{1,j} & \xi_{2,j} & \cdots & \xi_{j,j} & \cdots & \xi_{n,j} \\ ---- & ---- & --- & ---- & --- & ---- \\ \xi_{1,n} & \xi_{2,n} & \cdots & \xi_{j,n} & \cdots & \xi_{n,n} \end{bmatrix} \begin{Bmatrix} \eta_{j,1} \\ \eta_{j,2} \\ ---- \\ ---- \\ \eta_{j,j} \\ ---- \\ \eta_{j,n} \end{Bmatrix} \quad (9)$$

The square matrix, in equation (9), has an order equal (n x n), and it is in fact the transposed matrix of $[\xi]$, whose elements could be determined from the calibration process, as shwon before.

By solving equation (9), the elements of the row No. "j" of $[\eta]$ would be yielded. Similarly the other rows from No. (1) to No. (n) could be calculated, and hence $[\eta]$ of equation $(6)_b$ would then be defined.

As shown above, the determination of the matrix $[\eta]$ would require solving the matrix equation (9), "n" number of times. So the use of equation $(6)_b$ should not be sought unless the structure under consideration, would be investigated for load cases that exceeds the number (n), otherwise equation (5) would be more suitable.

METHOD OF APPLICATION

The process of selecting the locations "L", at which the strain is measured, is of vital importance. Not any point on the structure surface can serve this purpose. If one of the chosen locations happens to receive no strain from any one of the loading system components; e.g. when located at the free end of a transversally loaded overhanging beam; the obtained influence coefficient matrix $[\varepsilon]$ would then be singular. Mathematically, this means that the inverse of $[\varepsilon]$ does not exist, and consequently equation (5) would not be solved. Also equation $(6)_b$ could not be constructed, for the same reason.

From the practical point of view, it is not sufficient to assure that $[\varepsilon]$ is not singular, but also it should be a regular matrix, for getting accurate results from equations (5) or $(6)_b$, without extensive calculating effort. For this reason, the process of selecting the locations (L) has to be made in the light of both the structure statical system and the distribution of the subjected loads. Each strain measuring location should be assigned hypothetically to one load component. The best position for any strain location is that which experience maximum strain variation due to application of the load, to which it is assigned, and at the same time receives minimum strain due to application of the rest of the loading system. Both conditions can hardly be fulfilled, so a compromise has to be made in selecting the most favorite locations.

Generally speaking, for elements with supported ends, best results might be obtained by establishing the strain measuring locations at; or very close to, the assumed points of action of their corresponding assigned loads.

DEMONSTRATING EXAMPLE

To ascertain the ability of the prescribed method for application, several successful tests are performed on various cases. As an example, the case of the strip, shown in figure (3), is selected here for two reasons, other than its clarity. The first, is due to the fact, that this structure has not a finite supporting system. The second,is that the loading system is composed of concentrated and distributed loads, with steep variations and sudden changes. So, such a case is regarded as a challenging one, inspite of being a 2-dimensional problem.

The details of the tested structure, and the exerted loading, are given in figure (3). The subjected loading system is considered unknown and the yielded results of the loads are finally compared with them, for the sake of checking the method's reliability.

To compute the influence coefficient matrix $[\varepsilon]$; the strip is given two temporary supports at its ends, for carrying out the calibration process. The strip is hypothetically divided to twelve segments, shown in figure (3); and the resultant of the distributed load subjected to each segment is assumed, as a first trial, at the segment's middle point, where the strain measuring locations are chosen. The strain is measured at the strip lower fiber along the transverse direction. Matrix $[\varepsilon]$, which is a symmetrical matrix, is calculated as described before and its elements are given in Appendix I. The values of column matrix $\{\bar{\varepsilon}\}$, for strain developing under the action of the loading system are also given in that Appendix.

In figure (3), the subjected load system and the load values obtained from the first trial, are given. The small difference between the subjected and the computed load distributions is due to the approximation made, in the first trial, concerning the point of action of the resultants of the segments loads. By carrying out a second trial, using better corrected positions for the loads point of action, as obtained from the first trial, the yielded load results are found to be almost identical with the actual loads.

CONCLUSIONS

The loading system acting on any elastic structural member, including earth induced pressures, could be computed from the structure's strain informations. Number of locations on the structure surface are selected, for measuring the needed strain data. A calibrating-like process is to be performed on the structure to establish the relationship between the subjected loads and the developed strain at the chosen locations. The required load values are yielded from the solution of a matrix equation, whose elements are, either measured or computed strain values. The proposed method is tested by applying it on several cases, and found very efficient for application.

ACKNOWLEDGEMENT

Thanks are due to Engineer M.H. Khalifa, of Egyptian Academy of Scientific Research and Technology, for running this work computer programmes, in Cairo University, Computation Centre.

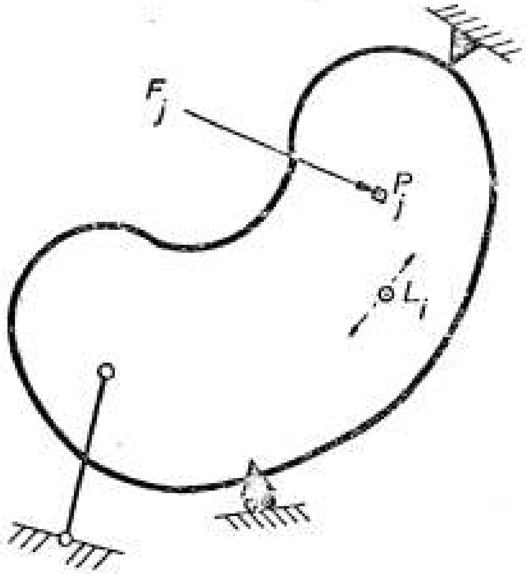

Fig. 1 - Generalized body acted by a single load

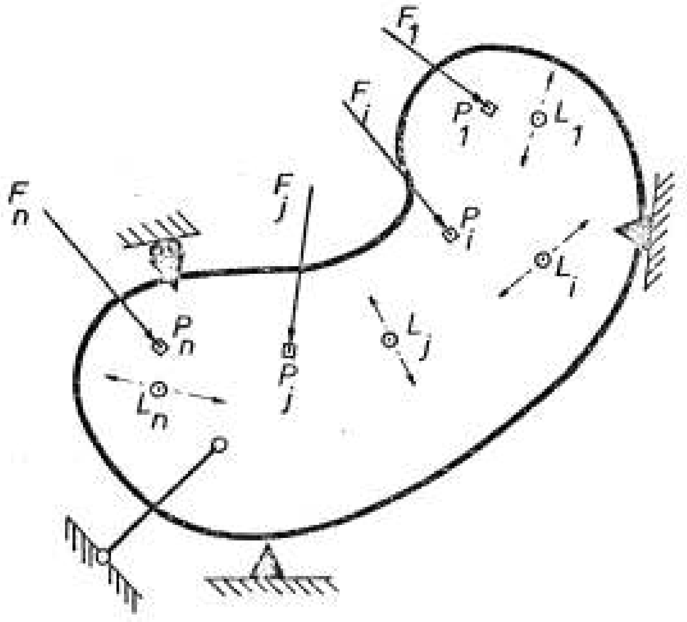

Fig. 2 - Generalized body acted by a group of loads

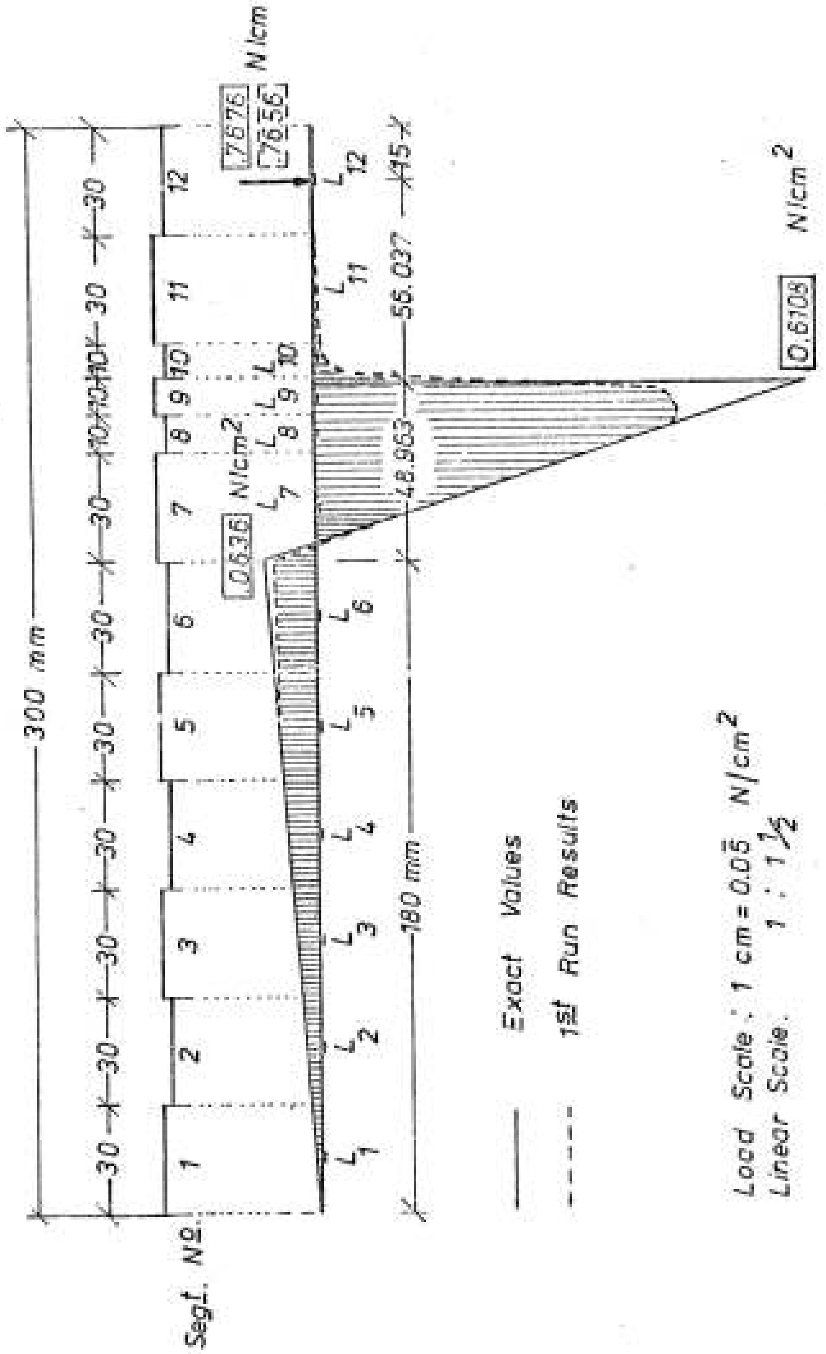

Fig. 3 - Studied Example

APPENDIX I- <u>EXAMPLE DATA</u>

Tested plate thickness = 0.76 mm

E.I = 7.168×10^{3} N x mm^{2} (per 1 mm breadth)

Coefficient Matrix Equation; $[\xi]$; computed due to point of load action at the middle of segments, shown in figure (3); reads:-

$$
= \begin{bmatrix}
755 & 675 & 596 & 516 & 437 & 357 & 278 & 225 & 199 & 172 & 119 & 40 \\
 & 2030 & 1790 & 1550 & 1310 & 1070 & 834 & 675 & 596 & 516 & 357 & 119 \\
 & & 2980 & 2580 & 2180 & 1790 & 1390 & 1125 & 933 & 860 & 596 & 199 \\
 & & & 3610 & 3060 & 2500 & 1950 & 1575 & 1390 & 1205 & 834 & 278 \\
 & & & & 3930 & 3220 & 2500 & 2025 & 1790 & 1549 & 1070 & 357 \\
 & & & & & 3930 & 3060 & 2475 & 2180 & 1893 & 1310 & 437 \\
 & & & & & & 3610 & 2925 & 2580 & 2237 & 1550 & 516 \\
 & & & & & & & 3225 & 2846 & 2466 & 1708 & 569 \\
 & & & & & & & & 2980 & 2581 & 1790 & 596 \\
 & & & & & & & & & 2695 & 1867 & 622 \\
 & & & & & & & & & & 2030 & 675 \\
 & & & & & & & & & & & 755
\end{bmatrix}
$$

The column matrix $\{\bar{\xi}\}$ reads :—

$$
\{\bar{\xi}\} = \begin{Bmatrix} 0 \\ 3 \\ 13 \\ 36 \\ 77 \\ 140 \\ 227 \\ 256 \\ 241 \\ 203 \\ 122 \\ 0 \end{Bmatrix}
$$

The computer programme used for solving matrix equation (5) reads,

```
      MASTER(HATEM)
C     GAUSS ELIMINATION METHOD
C BY ICL SUBROUTINE
      DIMENSION A (20,20),Y(20,20),X(20)
      DIMENSION AA(200),BB(14),W(3)
      DIMENSION AX(2,20),CH(14)
  999 READ(1,9)N,M
      IF(N.EQ.0)STOP
      IFLAG=0
      READ(1,10)((A(I,J),J=1,N),I=1,M)
      READ(1,10)((AX(I,J),J=1,N),I=1,2)
  802 DO 77 I=1,M
      DO 76 J=1,M
      KA=I+M*(J-1)
   76 AA(KA)=A(I,J)
      BB(I)=(-A(I,N)
   77 CONTINUE
      WRITE(2,222)
  222 FORMAT(////5X,'INPUT DATA'/5X,10(1H-)///)
      DO 67 L=1,M
   67 WRITE(2,69)(AL,J),J=1,N)
   69 CALL FPMGESOL(M,1,0.0001,AA(1),BB(1), W(1),
     +DET,IRANK, NRR)
      WRITE (2,71)
      WRITE(2,72)(KB,BB(KB),KB=1,M)
      DO 900 I=1,12
      CH(I)=-A(I,13)
      DO 901 J=1,12
  901 CH(I)=CH(I)-BB(J) A(I,J)
  900 CONTINUE
      WRITE(2,800)(KI,CH(KI),KI=1,12)
  800 FORMAT(/,(/T10,'CH(',I2,')=',E14.8))
      IF(IFLAG.EQ.1)GO TO 701
      DO 801 I=1,13
      A(8,1)=AX(1,I)
  801 A(10,I)=AX(2,I)
      IF(IFLAG.NE.0)STOP
      IFLAG=1
      GO TO 802
      GO TO 999
```

```
71   FORMAT(////,T40,'RESULTS OF SUB. PPMGESOL',
    +/T40,24(1H-),///)
72   FORMAT(/T10,'X(',I2,') = ',E14.8)
 9   FORMAT(2I3)
10   FORMAT(13F6.1)
     STOP
     END
```

APPENDIX II. NOTATIONAL SYMBOLS

$E.I$ = flexural stiffness of loaded member,
= Modulus of Elasticity x Moment of Inertia of member,

$F_1, F_2 \ldots F_n$ = concentrated loads; forces; acting at points P_1, P_2 ... P_n, respectively,

$f_1, f_2 \ldots f_n$ = scalar magnitude of loads $F_1, F_2 \ldots F_n$, respectively,

i, j = subscripts referring to two different digits that can be assigned to any number between "1" and "n",

$L_1, L_2 \ldots L_n$ = locations of strain measurement, each defines a point and a direction along which the state of strain is measured,

N = axial tension along the neutral axis,

n = the number of loads "F" acting on a member. Also the number of locations "L" selected on that member,

$P_1, P_2 \ldots P_n$ = points of action of loads $F_1, F_2 \ldots F_n$, respectively,

P = applied transverse load per unit length,

w = transverse deflection of neutral axis,

x = the distance along the neutral axis,

$\varepsilon_{i,j}$ = normal strain developed at location L_i, due to a unit load applied at point P_j in direction of load F_j,

$\bar{\varepsilon}_{i,j}$ = normal strain developed at location L_i, due to load F_j,

$\bar{\varepsilon}_i$ = normal strain developed at location L_i, due to the load group $F_1, F_2 \ldots F_n$.

BIBLIOGRAPHY

Enger, M.L., (1916). "High Unit Pressure Found in Experiments on Distribution of vertical loading". Engineering, Vol. 73, p. 106.

Evans, W.H., (1940). "Dynamometer for Measuring Ground Pressures". Engineering, Vol. 149, p. 433.

Goldbeck, A.T. & Smith, E.B., (1916). "An Apparatus for Determining Soil Pressures". Proc., A.S.T.M., Vol. 16, part 2, p. 309.

Martin, H.C., (1966). "Introduction to Matrix Methods of Structural Analysis". McGraw-Hill Book Company.

Mackey, R.D. & Creighton, P., (1965). "Pressure cell using Semi-Conductor Gauges". Engineering, Vol. 199, No. 5165, P. 513.

Redshaw, S.C., (1954). "A miniature Earth Pressure Cell". Jour. of Scientific Instruments, Vol. 31, P. 467.

Taylor, D.W., (1947). "Review of Pressure Distribution Theories, Earth Pressure Cell Investigation and Pressure Distribution Data". U.S. Waterway Exprimental Station, S.M. Fact Finding Survey, Part II.