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THE INFLUENCE OF DEPTH AND FRICTION ON THE TIDAL MOTION IN CLOSED RECTANGULAR BASINS.

By

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ABSTRACT

The Mediterranean Sea has been simplified in its shape as consisting of two closed rectangular basins with constant depth. The basins were assumed to be either separated at Tunis Strait by a barrier or connected together through an opining of 2° width. A numerical model developed by Soliman *<u>et</u> <u>al</u>. (1993) has been tested several times for the fore mentioned shapes with constant depths which varied between 100 m and 2000 m. Special attention was paid to the pronounced deviations in the amplitudes of the two basins for each shape. The co-tidal lines showed some peculiarities between the different depths. Considerable increase in amplitude was observed at the different conditions in each basin which revealed the approach of the basin from the resonance conditions, i.e. the increase in amplitude is very large when the frequency of the tidal motion is close to one of the natural frequencies of oscillation of the basin. Therefore, the length of the basin as well as its depth significantly influence the tidal characteristics.*

Moreover, it is evident from the results that friction does not only affect the amplitude but also the phase distribution. Generally, it develops the tidal patterns; and limits the increase of the amplitude near resonance.

The Coriolis parameter and the tidal potentials may cause the transverse oscillations which lead to the formation of the amphidromic system. The influence of the Coriolis parameter may decrease with increasing water depth and can be ignored in deep sea basins.

The free period may be regarded as a control factor for both the type of rotation of the amphidromic system and its associated tidal ranges.

INTRODUCTION

The term "tidal" as applied to waves, has been used in various senses. It is mostly confined to gravitational oscillations having the characteristic feature of the oceanic tides produced by the action of the sun and moon. Upon this restriction, many investigator have written about this subject (Laplace, 1775; Airy, 1842 & 1845; Thomson, 1875; Darwin, 1883; Doodson, 1921, 1935 & 1937; and Lamb, 1932).

During the first three decades of this century, solutions to the tidal equations were given either by expanding in series of suitable functional systems or by solving an infinite system of equations (Goldsbrough, 1913 & 1914; Proudman, 1916 & 1931; Doodson, 1935 & 1937; and Doodson, 1958). Defant (1919) was the first to compute tidal distributions in one dimensional sea by means of numerical integration. The method has been improved by Hansen (1948, 1956& 1966). Generally, the problem concerning gravity waves in its broad sense in rectangular basins has been treated by many investigators (Taylor, 1922; Lamb, 1932; Defant, 1961; Krauss and Magaard, 1962; Platzmann and Rao, 1964; Platzmann et al., 1981; and Rao, 1966). The influence of depth on the tidal motion has not been directly considered. Laplace (1775) found that, on considering the tides for an ocean of constant depth covering the whole earth, the tide at the equator is 180 degrees out of phase with the equilibrium tide and 7.4 times higher in amplitude for a depth of 7260 feet. It is still 180 degrees out of phase, but only 1.8 times higher for an ocean 14, 520 feet deep. For an ocean 29,040 feet deep the tide is in phase with the equilibrium tide and 11.2 times higher. Doodson (1937) mentioned that for an ocean 16,130 feet deep there are three amphidromic points in the northern hemisphere. With the relatively slight change to 15, 280 feet one whole amphidromic system disappears.

Eid et al. (1993) noticed that there are clear differences in the tidal ranges between rectangular basins with constant depths of 500 m and 1500 m. To get an explanation for such variabilities, the present work has been conducted. In addition, significant differences have been resulted in examining the same model on the Mediterranean Sea with its natural depths and coastal boundaries with grid size 1° X 1° (Abdallah et al., 1993). To understand the reasons for getting such differences, as well as to examine the influence of friction, coastal and bottom configurations and the Coriolis force, the same model has been tested again in rectangular basins with different depths and different friction coefficients.

The model

Hansen model, developed by Soliman et al. (1993) has been used. The vertically integrated differential equations that formulated to the difference equations are as the followings:

$$\frac{\delta u}{\delta t} - fv + ku + g \frac{\delta \xi}{\delta x} X = 0 \qquad \ldots\ldots (1)$$

$$\frac{\delta v}{\delta t} + fu + kv + g \frac{\delta \xi}{\delta y} Y = 0 \qquad \ldots\ldots (2)$$

$$\frac{\delta}{\delta t} + \frac{\delta (Hu)}{\delta x} + \frac{\delta (Hv)}{\delta y} = 0 \qquad \ldots\ldots (3)$$

where:

x,y : cartesian co-ordinates in the east and north direction respectively,

t : time,

ξ : water elevation of the sea surface,

u,v : components of the depth mean current in x & y directions respectively,

H : total depth of water,

f : Coriolis parameter,

k : coefficient of bottom friction in the linear form,

g : acceleration of the earth's gravity,

X,Y : the components of the tide - producing force in east and north directions respectively.

A grid system of mesh size 1° X 1° was used (Fig. 1). The zero values for water elevation and horizontally current components have been assumed as initial conditions.

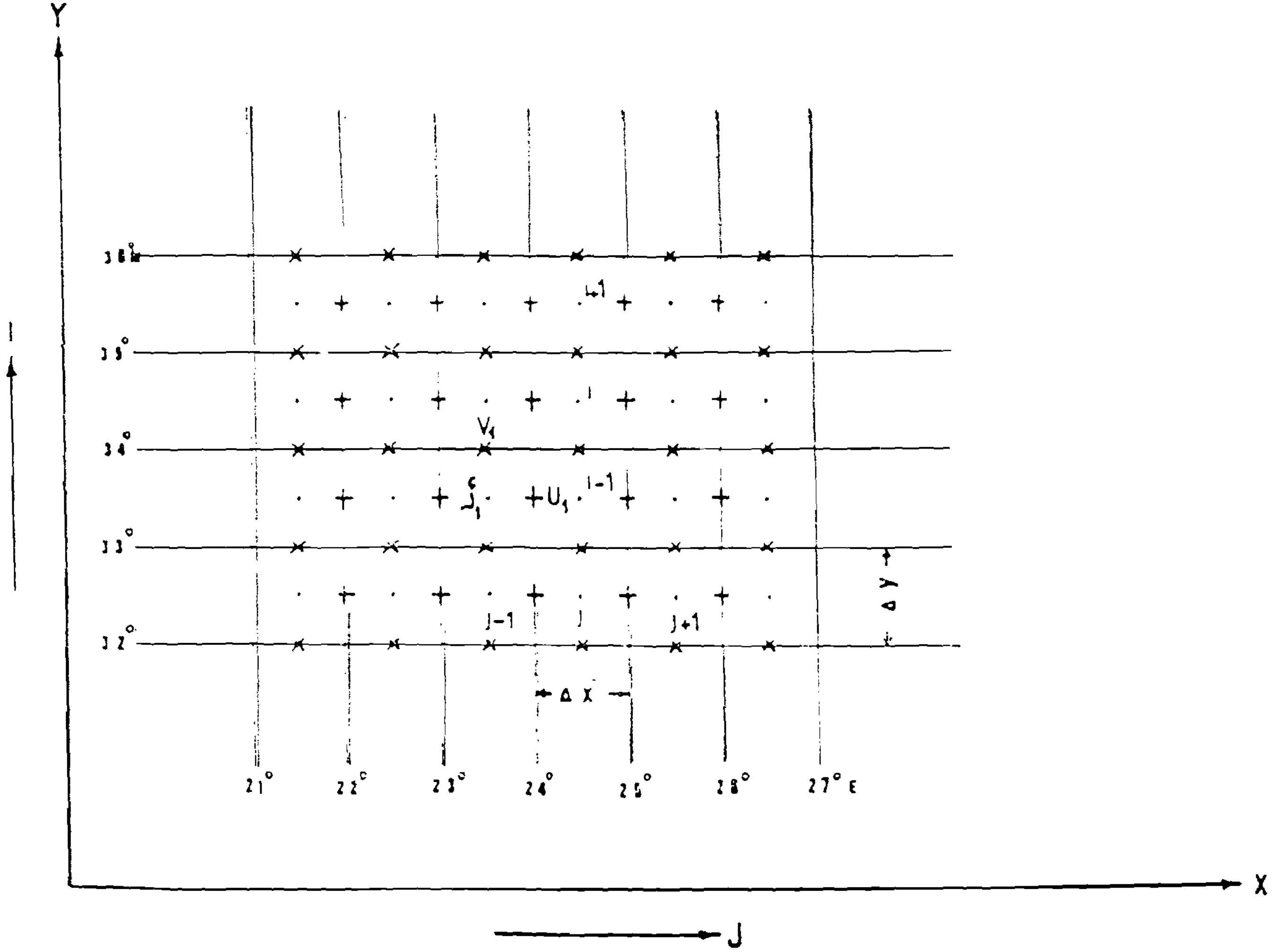

Figure 1: The grid system.

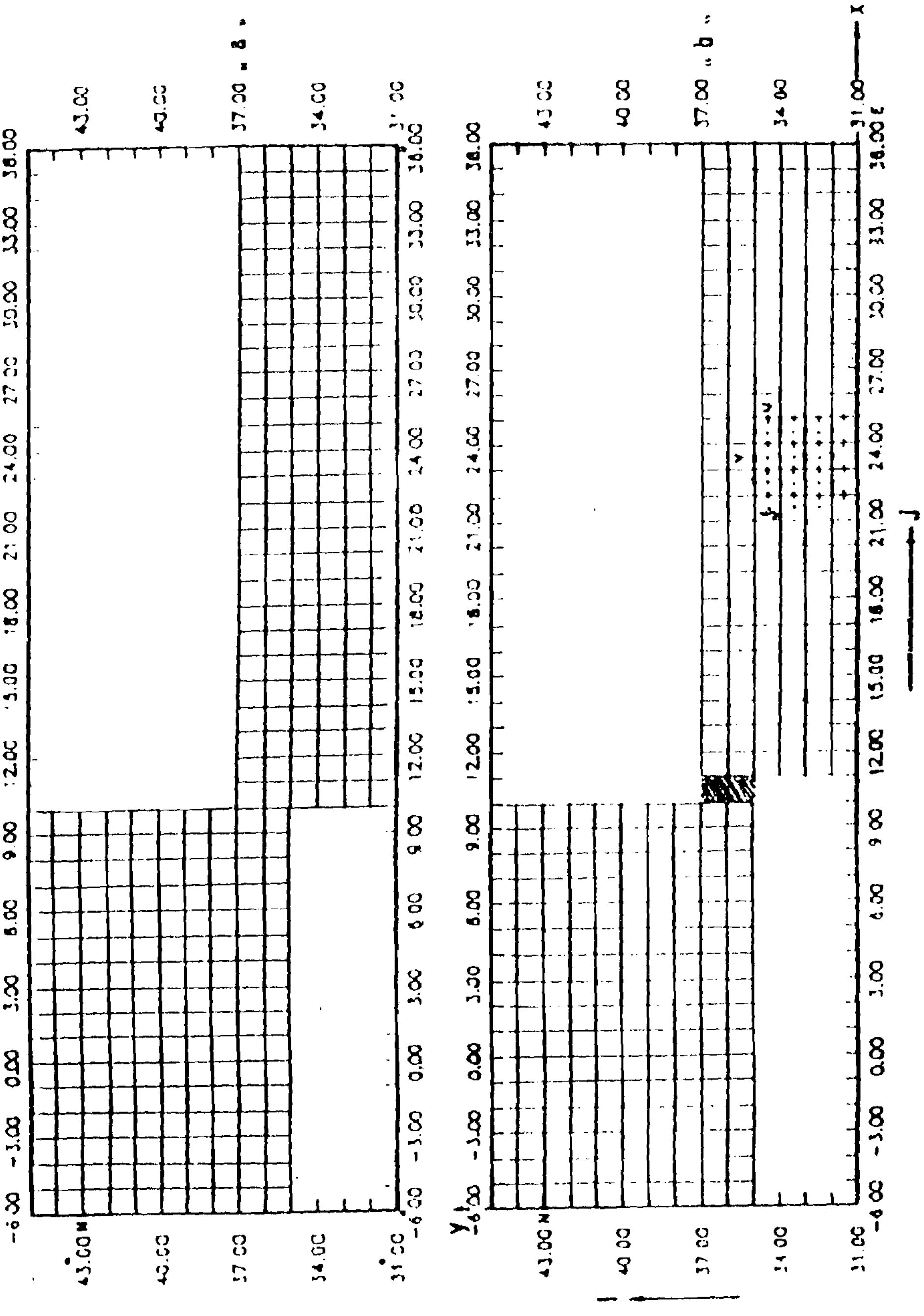

Figure 2: a- The sea as consisting of two connected rectangular basins.
b- The sea as consisting of two separate rectangular basins.

RESULTS AND DISCUSSION

The shape of the sea has been suggested as consisting of two rectangular basins with constant depth, connected together through an opening of 2o width (Fig. 2a).

To investigate the influence of co-oscillation of each basin on the other, the two basins have been separated at the line of connection (10° Long.) by a rigid barrier. Hence a new shape has been developed (Fig. 2b), which showed two separate basins (corresponding to the eastern and western basins of the Mediterranean).

The model has been applied for each shape assuming a linear relationship of friction coefficient of 10-5 & 10-6 sec-1.

Assuming friction coefficient of 10-6 sec-1, the model has been applied to the two cases of basins (Table 1) having constant depths of 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1500 & 2000 m in consequence (Fig. 3 and Fig. 4). On a non-rotating earth (f=0) and depths of 100, 300, 500, 700, 900 & 1500 m, the results were given in Figures (5 & 6). By considering the friction coefficient as 10-5 sec-1, the results obtained for the same series of depths were presented in Figures (7 & 8) which generally showed similar patterns as given in Figures (3 & 4) with relatively lower amplitudes (Table 2). These reveal that friction does not only influence the amplitude but also the phase distribution (Tables 1 & 2). It suppresses to a certain extent the increase in the amplitudes in case of resonance (Fig. 9).

These suppression in amplitudes in the case of the connected basins indicate that there exists a kind of co-oscillation between the two basins. i.e. in addition to the oscillations formed in one basin by the direct generating forces there is another component superimposed on the previous one caused by the co-oscillation of this basin with the other one. On the other hand, the type of tidal motions produced in the connected basins is not necessarily be identical with the oscillations derived in one rectangular basin having the same dimensions. However, the motion (deep connection greater than 800 m) appears as composed of a stationary wave which differs greatly from that produced in a closed rectangular basin with the same dimensions (Eid et al., 1993). Actually, the amplitudes in the last basin were magnified to large values. Such features may explain the extra-ordinary amplitudes observed in the Gulf of Gabes, where amplitudes of more than one meter are existing. This phenomenon does not experienced in any other location throughout the Mediterranean Sea. This reveals that, if the period of the free oscillation of the basin approaches from the forced period, the system will be very close to the condition of resonance and hence the tidal ranges will increase considerably.

Table (1): Amplitudes (in cm) and phaese (in degrees, of high tide after lunar transit at Greenwich) of computed M2-Independent Tides in a closed sea as consisting of two connected and separate rectanguler basins with friction coefficient of 10-6.

STATION	ABB.	TWO SEPARATE BASINS r=10-6				TWO CONNECTED BASINS r=10-6			
		CONSTANT DPTH							
		500 m		1500 m		500 m		1500 m	
		A(cm)	ø	A(cm)	ø	A(cm)	ø	A(cm)	ø
GIBRALTAR	GI	21.0	216	06.1	075	18.6	112	02.2	186
ALGER	AG	19.7	312	02.2	314	08.4	234	05.1	280
TUNIS	TU	05.5	210	12.6	071	05.7	229	07.9	355
SFAX	SF	04.8	226	12.5	068	01.2	210	10.2	004
GABES	GB	04.6	236	12.4	066	01.5	119	12.5	007
TRIPOLI	TL	05.4	258	10.9	063	02.9	086	13.1	010
BANGHAZI	BG	05.5	287	04.3	055	04.3	079	10.1	016
TUBRUQ	TQ	04.8	337	02.3	269	02.8	046	05.3	042
SALOM	SA	04.9	348	03.5	261	02.4	029	04.6	054
MARSA MATROH	MM	05.3	008	05.7	253	02.4	341	03.3	091
ALEXANDRIA	AX	05.6	023	07.9	250	03.6	309	04.4	125
PORT-SAID	PD	05.7	043	10.6	245	05.9	289	06.7	154
BEIROT	BE	05.2	067	12.3	240	06.7	278	09.1	162
ATHENS	AH	03.2	156	00.5	038	08.0	083	03.6	052
RHODES	RH	03.4	102	03.2	249	05.3	057	03.8	100
LA-SPEZIA	LS	21.5	044	05.6	243	24.4	281	08.5	223
GENOVA	GE	21.5	044	05.6	243	24.4	281	08.5	223
IMPERIA	IM	21.1	047	05.2	242	23.4	283	08.2	223
MONACO	MO	20.3	053	04.6	241	21.6	287	07.7	222
NIECE	NI	20.3	053	04.6	241	21.6	287	07.7	222
CANNES	CN	20.3	053	04.6	241	21.6	287	07.7	222
TOULON	TO	19.2	060	03.9	238	19.5	291	07.4	219
MARSEILLE	MS	18.2	069	03.2	234	17.1	297	07.2	215
BARCELONA	BA	17.3	107	01.6	197	09.9	332	06.6	206
MALTA	MA	05.6	220	10.2	069	18.6	145	08.2	019

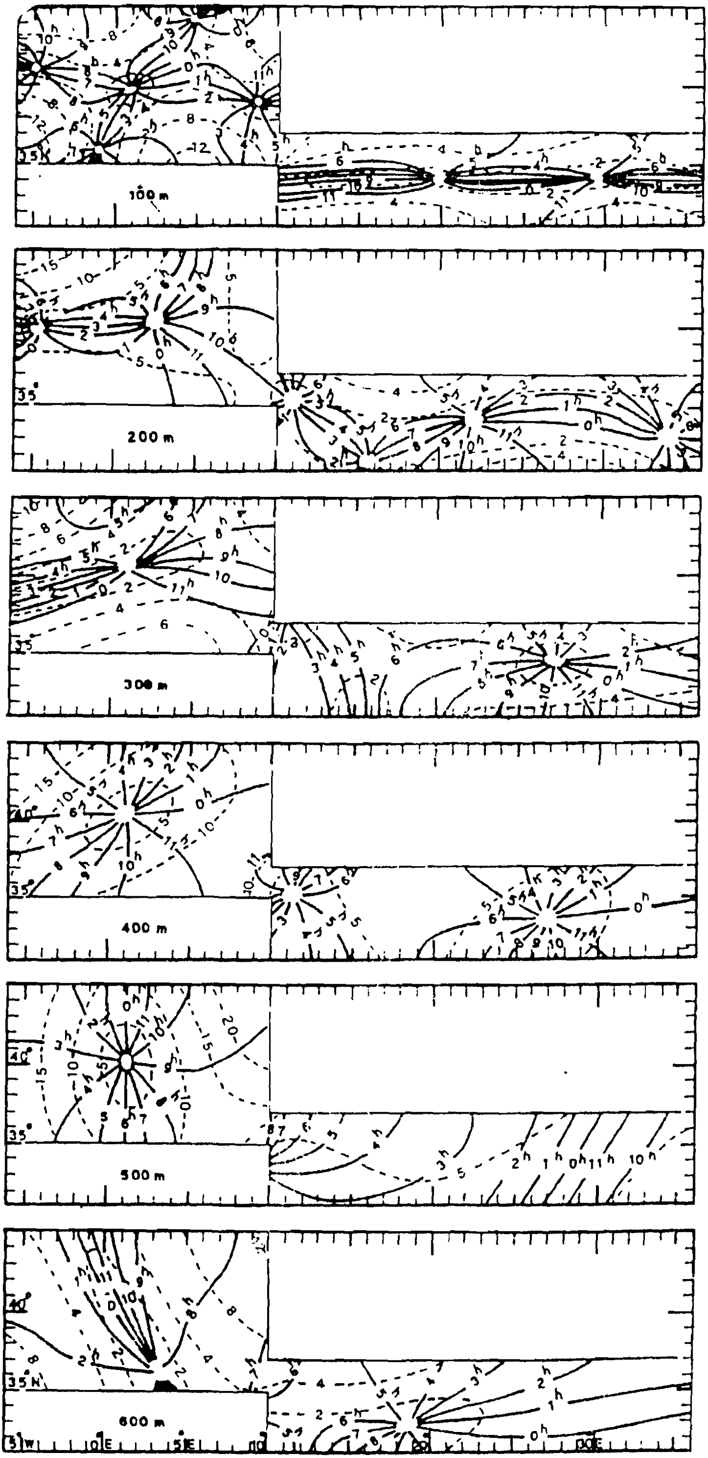

Figure 3-a: M2- Independent Tide in the connected basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

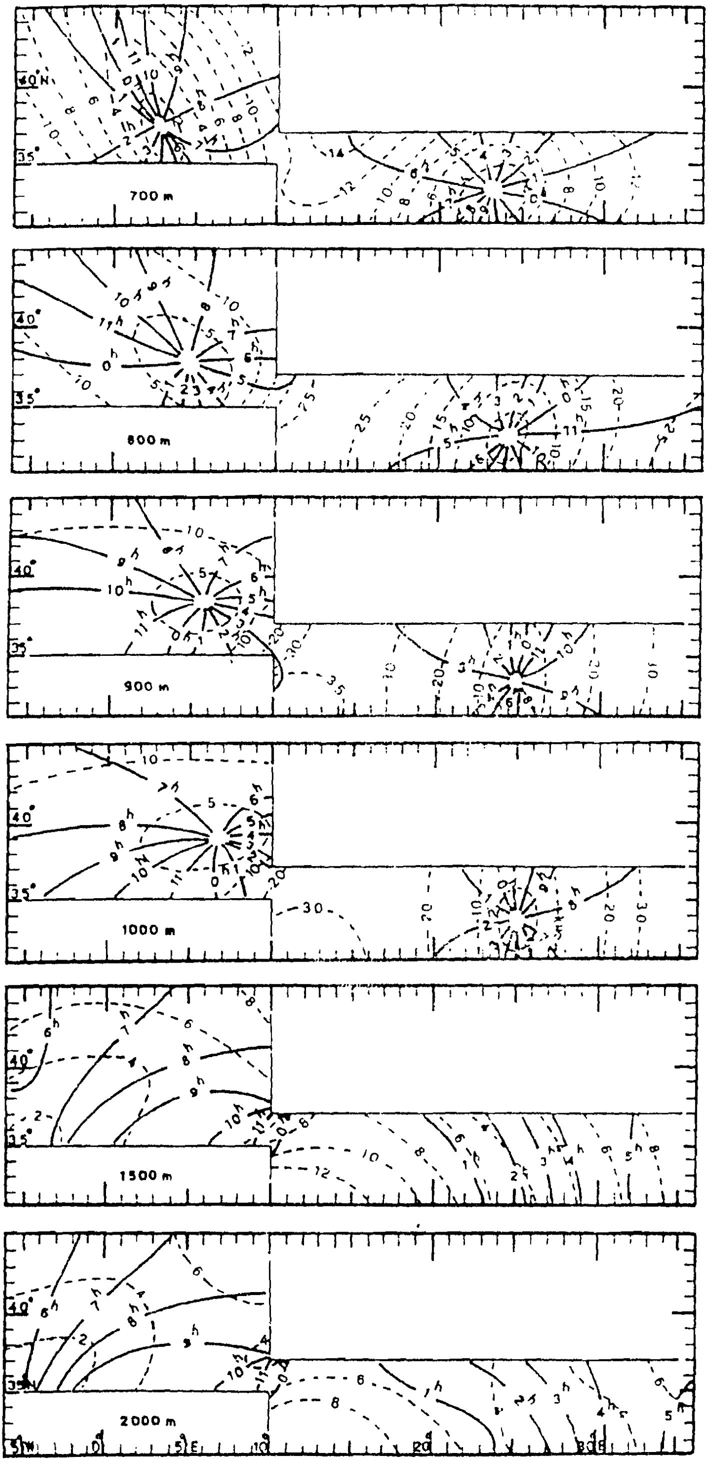

Figure 3-b: M2- Independent Tide in the connected basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

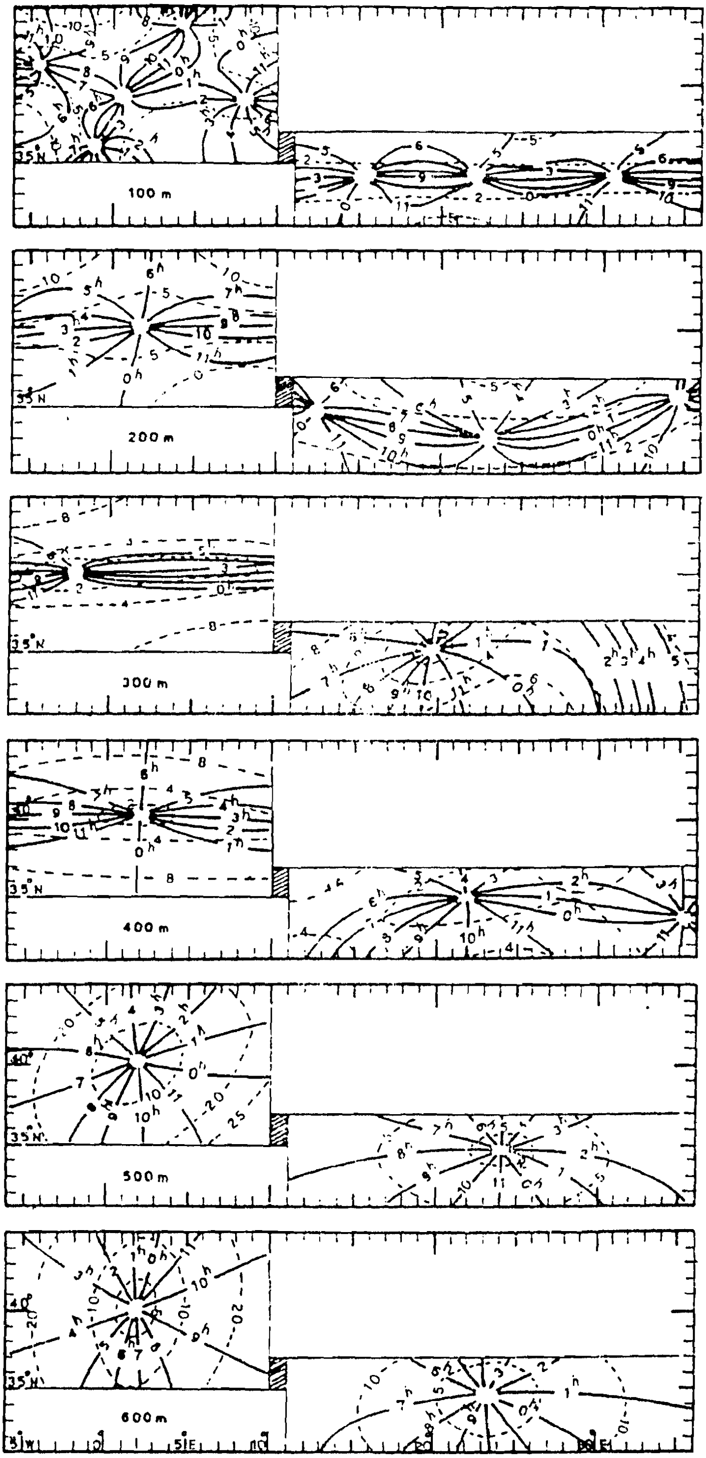

Figure 4-a: M2- Independent Tide in the separate basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

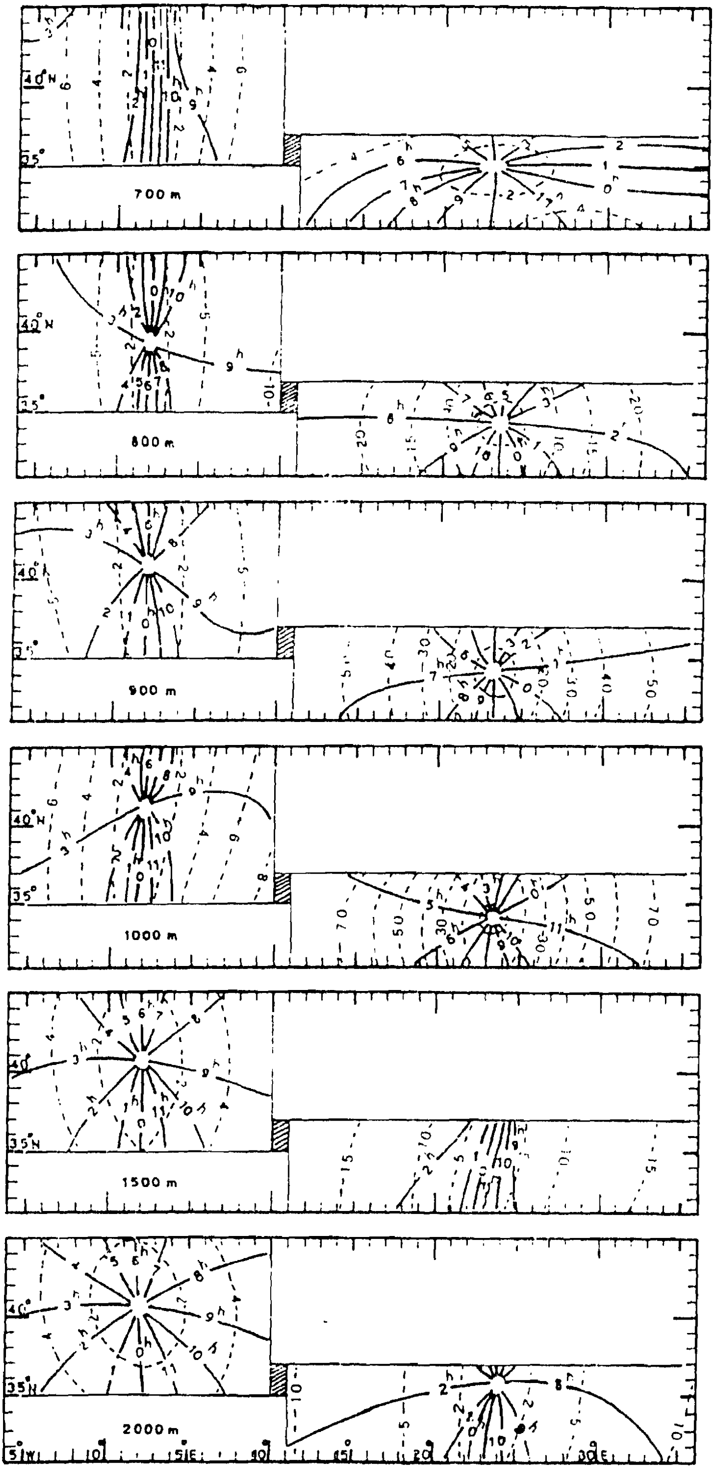

Figure 4-b: M2- Independent Tide in the separate basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

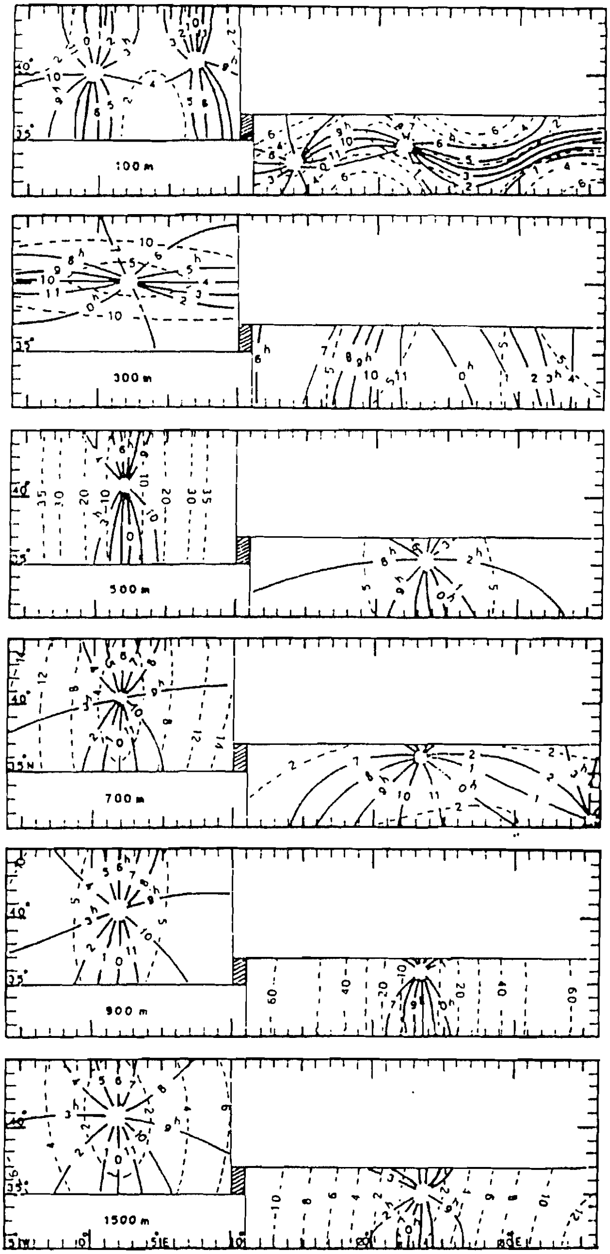

Figure 5: M2- Independent Tide in the separate basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. on a non-rotating earth, i.e. f= o. The co-range are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

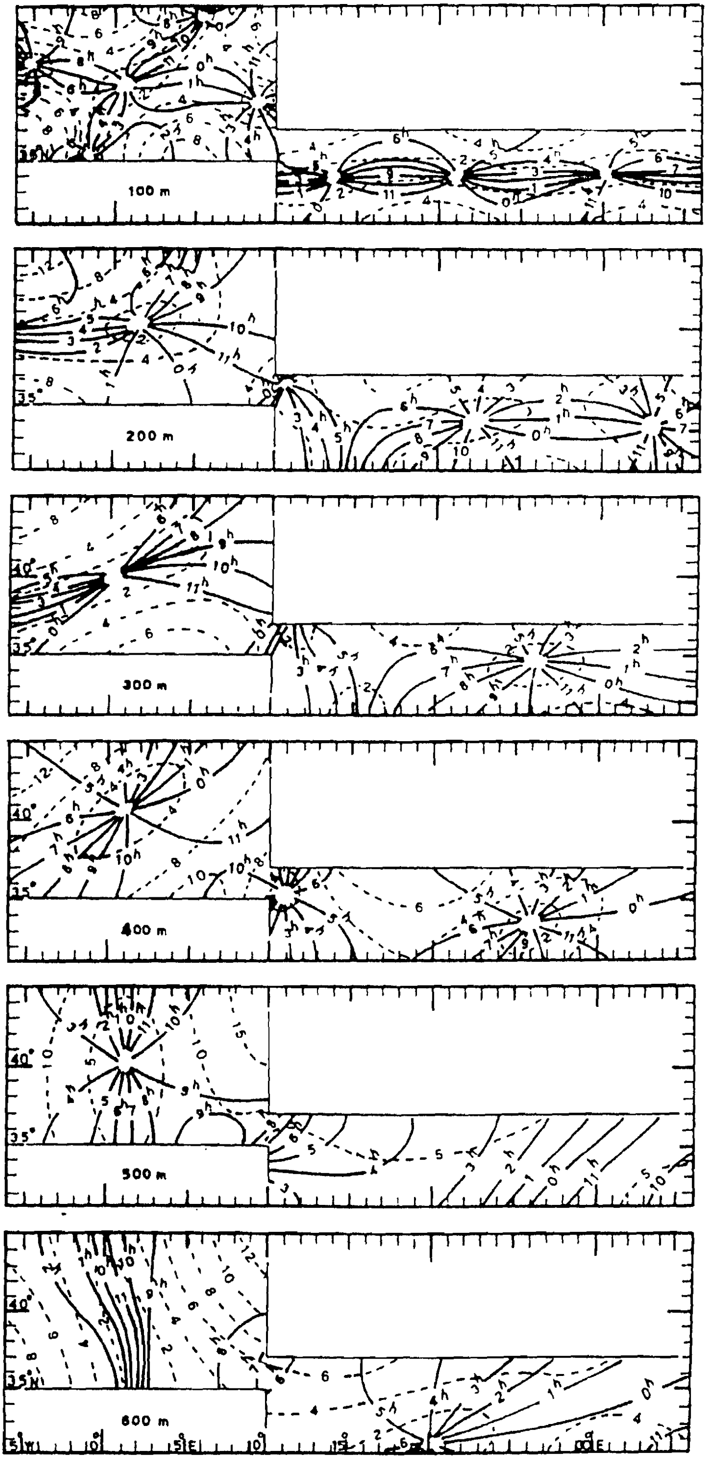

Figure 6a: M2- Independent Tide in the connected basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. on a non-rotating earth, i.e. f= o. The co-range are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

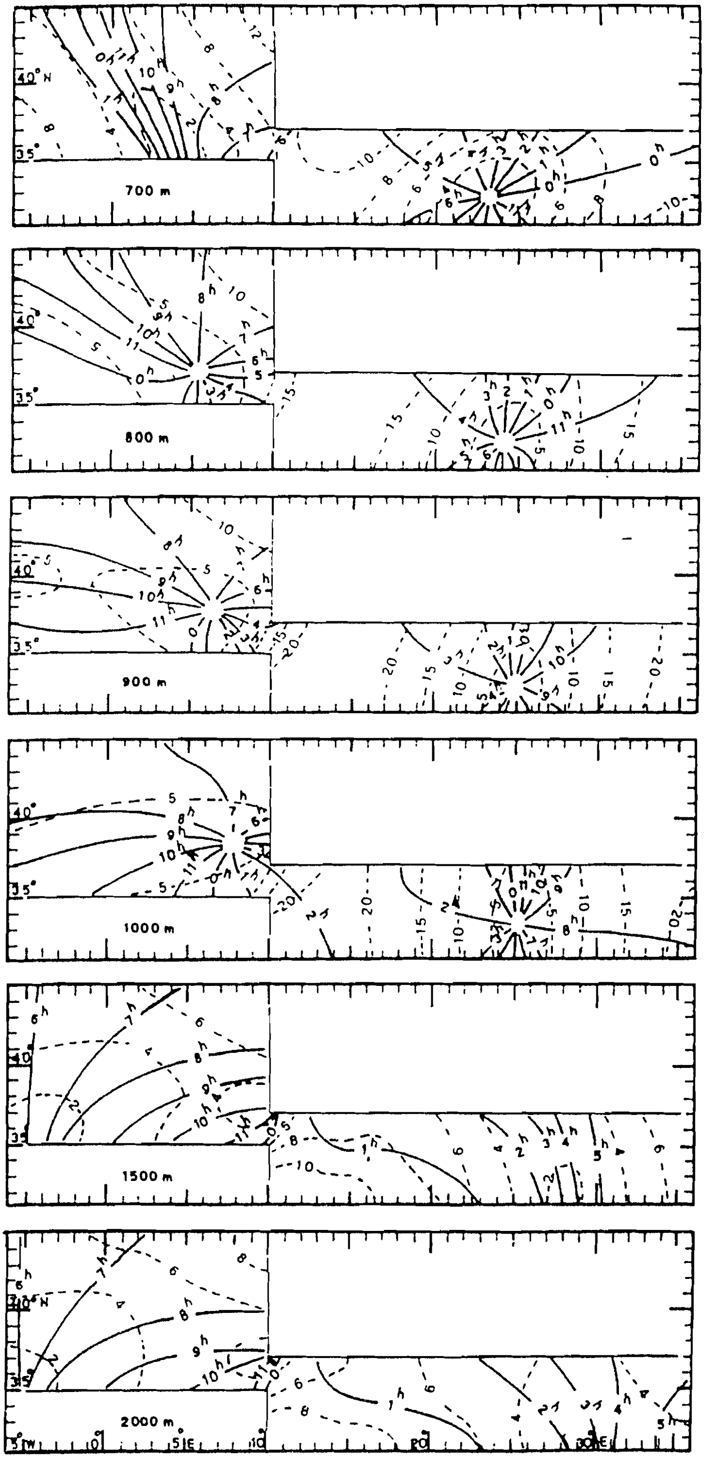

Figure 6b: M2- Independent Tide in the connected basins at different depths and with friction coefficient of 10^{-6} sec^{-1}. on a non-rotating earth, i.e. f= o. The co-range are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

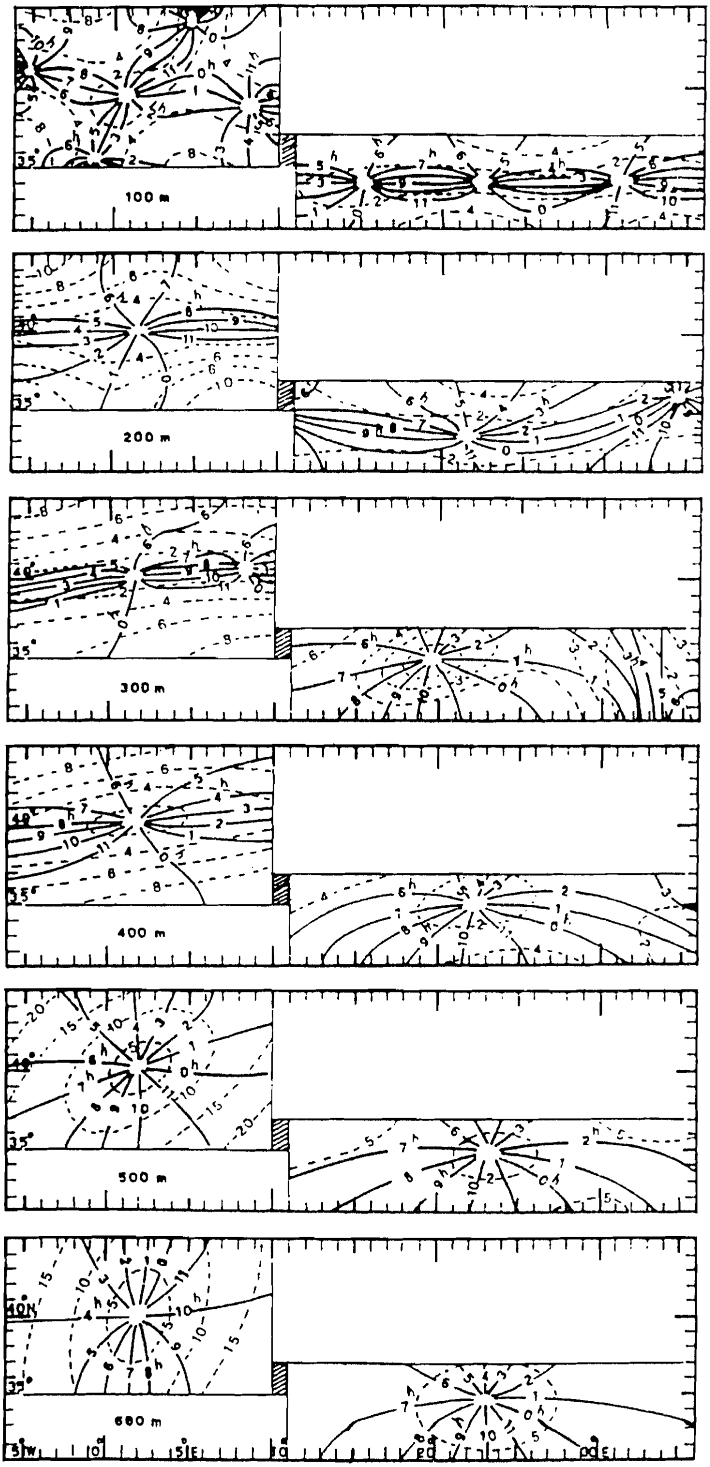

Figure 7a: M2- Independent Tide in the separate basins at different depths and with friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

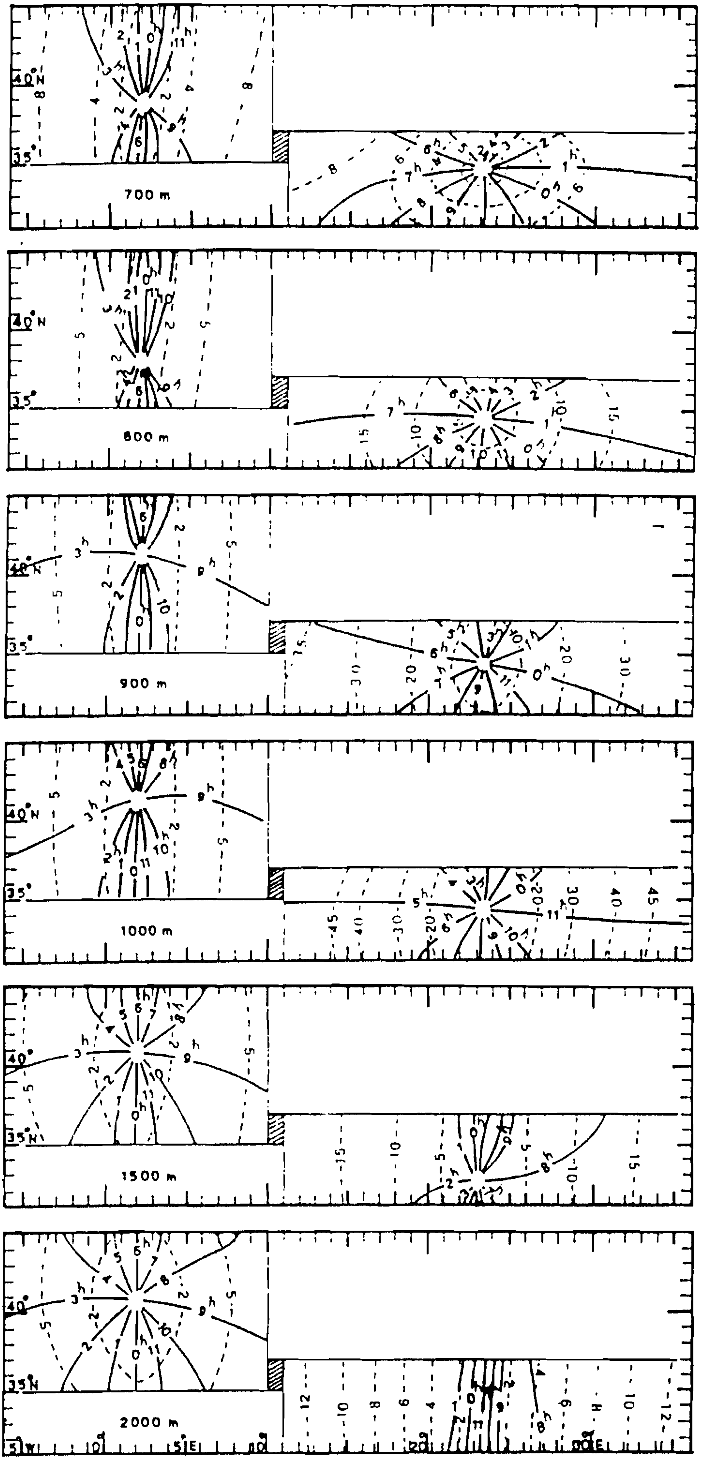

Figure 7b: M2- Independent Tide in the separate basins at different depths and with friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

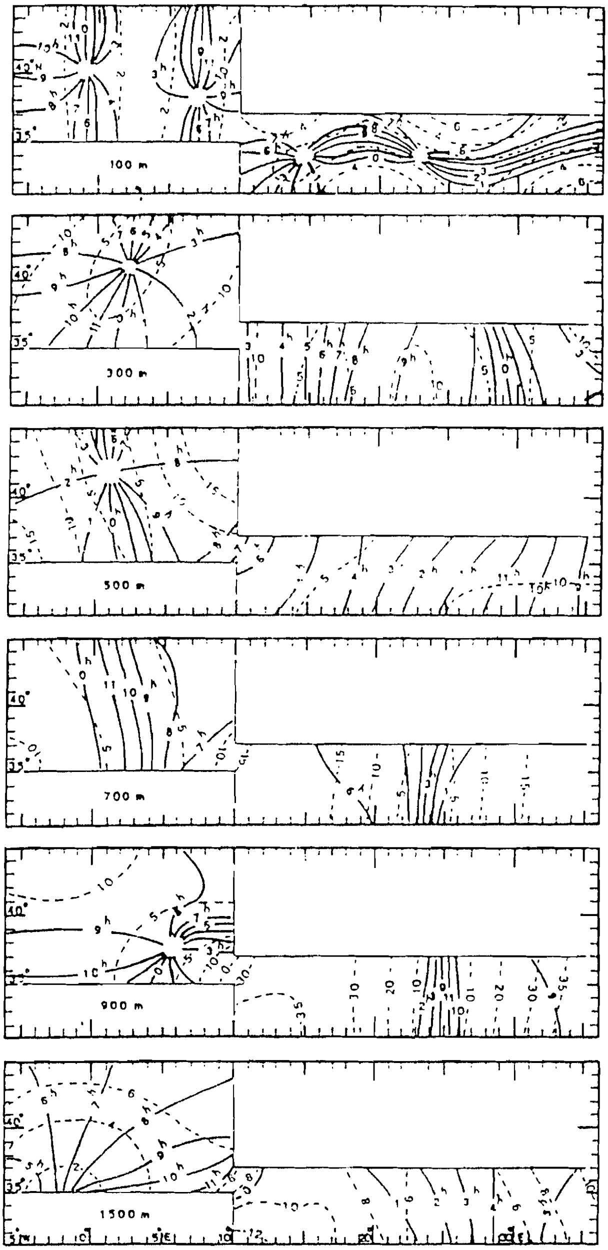

Figure 8: M2- Independent Tide in the connected basins at different depths and with friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

Table 2 : Amplitudes (in cm) and phases (in degrees, of high tide after lunar transit at Greenwich) of computed M2- tides in a closed sea as consisting of two separate and connected rectangular basins with friction coefficient of 10-5.

STATION	ABB	TWO SEPARATE BASINS r=10-5				TWO CONNECTED BASINS r=10-5			
		CONSTANT DEPTH							
		500 m		1500 m		500 m		1500 m	
		A(cm)	ø	A(cm)	ø	A(cm)	ø	A(cm)	ø
GIBRALTAR	GI	13.3	212	06.0	077	12.9	117	01.4	175
ALGER	AG	14.7	313	02.2	321	06.3	254	04.4	285
TUNIS	TU	06.0	185	18.1	061	03.8	232	06.5	009
SFAX	SF	04.9	196	18.2	060	00.6	164	08.7	016
GABES	GB	04.4	205	18.3	059	01.8	107	10.9	018
TRIPOLI	TL	04.7	234	16.6	058	02.5	096	11.3	019
BANGHAZI	BG	05.0	276	07.3	060	02.8	094	08.7	024
TUBRUQ	TQ	04.9	326	02.4	231	01.6	030	04.4	038
SALOM	SA	05.0	335	04.4	234	01.8	005	03.6	046
MERSA MATROT	MM	05.3	352	08.1	235	02.7	332	01.6	078
ALEXANDRIA	AX	05.4	005	11.5	236	04.0	316	02.1	143
PORT-SAID	PD	05.0	022	15.7	235	05.6	302	04.6	180
BEIROT	BE	03.9	044	18.1	233	05.8	292	06.9	183
ATHENS	AH	02.7	133	01.7	333	06.8	092	03.7	053
RHODES	RH	03.1	081	05.7	251	04.6	064	02.7	091
LA-SBEZIA	LS	13.5	043	05.4	245	17.5	284	08.2	222
GENOVA	GE	13.5	043	05.4	245	17.5	284	08.2	222
IMPERIA	IM	13.3	047	05.0	244	16.6	286	07.9	221
MONACO	MO	12.9	052	04.3	243	15.2	289	07.5	220
NIECE	NI	12.9	052	04.3	243	15.2	289	07.5	220
CANNES	CN	12.9	052	04.3	243	15.2	289	07.5	220
TOULON	TO	12.4	060	03.7	241	13.5	294	07.1	218
MERSEILLA	MS	11.9	071	03.0	237	11.7	300	06.9	215
BARCALONA	BA	12.3	109	01.4	194	06.1	337	06.2	208
MALTA	MA	05.4	208	15.3	059	05.9	148	07.5	036

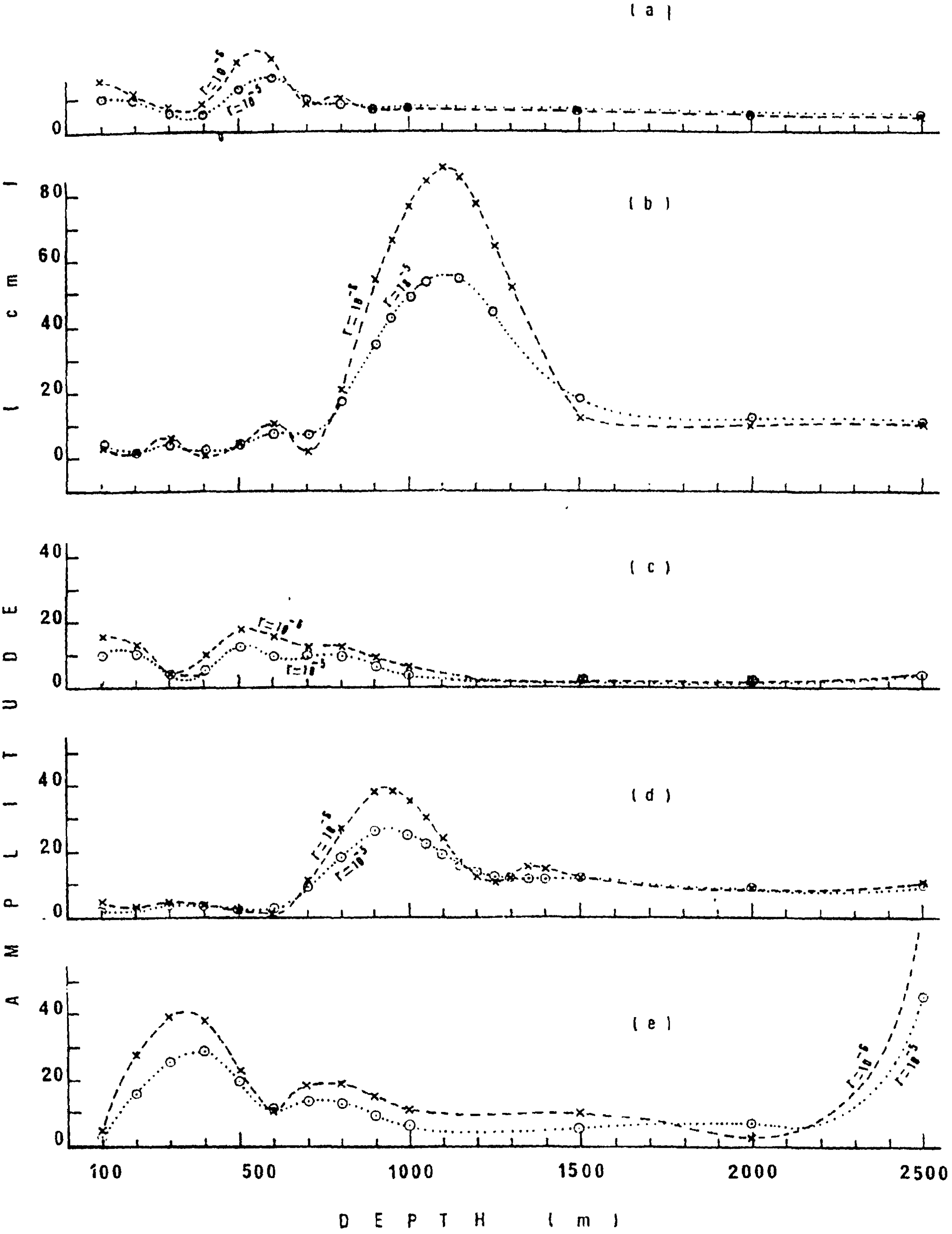

Figure 9: The variation of Sea level with depth in both basins.

With increasing the depth of the basin, the number of amphidromies has decreased where one amphidromy mostly appeared in each basin for great depths.

The co-ranges described different variety features with depth in the case of two separate basins. In the western basin they showed firstly a decrease tendency with increasing depth then, they have started to increase to reach large values at 500 m depth as a result of approaching from resonance. For great depths of more than 900 m, the amplitudes fall again showing nearly constant values. In the eastern basin, high amplitude values were observed at a depth of 1000 m where they started to decrease again. Similar patterns were watched in the case of the connected two basins.

Generally, for depths greater than 600 m, the motion is simply represented by a standing wave with a nodal line nearly situated at the center of each basin. For depths greater than 900 m, the western basin shows a cum sole amphidromy, which indicating that for great depths the influence of the earth's rotation may be ignored. The type of motion obtained in the western basin is nearly similar in the two cases. Comparing these patterns with that obtained by Sterneck (1922) to compute M2-tide in the Black Sea, a good agreement was found both for amplitudes and phases although the depth of the basin was assumed to be constant in the present study.

The tidal distributions showed that, for relatively small depths, the amphidromy often changes its rotation sense from one depth to another, i.e. from cum sole amphidromy to contra solem one and vice versa. It is well known that, in the absence of earth's rotation, the amphidromy is anticyclonic while it is cyclonic on rotating earth. The rotation of the earth causes strong transverse oscillations which transform the nodal lines into amphidromies contra solem. Applying the tidal forces on the Black Sea as a closed basin and taking into account the rotation of the earth, Defant (1961) found that the distribution of the semi-diurnal tides showed an amphidromy rotating to the right (cum sole). He explained such reverse rotation as a result of considering the oscillations caused by the Coriolis force to be negligible.

The direction of rotation can be easily investigated from figure 9, which representing the water elevations in both basins as a function of depth. The motion in the basin will indicate only the tidal oscillations if the natural frequency of the basin is very small compared to the tidal forces. Actually, this feature is difficult to exist in nature, since the natural boundaries, either the bottom or the coastal topographies or both, are too complicated to make the estimation of the free oscillations period also difficult. Generally, Fig. 9 can help greatly in understanding the behavior of the tidal motion in each basin.

CONCLUSIONS

From the above discussions and the distributions of the tidal lines the following are concluded:

1- Friction does not only influence the amplitude but also the phase distribution. It develops only the tidal oscillations.

2- In the shallow depths, the resolution is very high, hence more than one amphidromy is expected in the basin. With increasing the depth, the number of amphidromies decreases where one amphidromy is formed.

3- The direction of rotation of the existing amphidromy changes from one attitude to another by increasing the depth of the basin.

4- The influence of the rotation of the earth is found to be negligible in basins with great depths.

5- The influence of the Coriolis parameter may cause oscillations which are perpendicular to the direction of the tidal wave and therefore may develop into amphidromic waves which completely changing the original character of the phenomenon. The transverse oscillations may also be produced by the vertical component of the tidal forces. The increase in amplitude is very large (i.e. resonance occurs) when the frequency of forcing is close to one of the natural frequencies of oscillation.

6- If connection exists between any two basins, co-oscillating motions will take place between them.