Bull. Nat. Inst. Ocn. & Fish., A.R.E. 1993. (19): 21-35

NUMERICAL INVESTIGATION OF M2-TIDES IN THE MEDITERRANEAN, AS A CLOSED RECTANGULAR BASIN.

By

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Key Words: M2-Tides, Mediterranean.

ABSTRACT

A two dimensional numerical model of Hansen type is used to investigate the influence of the basin shape and water friction on the tidal motion in a closed basin. Different shapes have been considered as a single rectangular basin, L-shaped and two connected rectangular basins. Each parameter contributes a significant influence on the tidal characteristics. Moreover, the tidal regime in each basin is affected with the adjacent basin through compiling co-oscillations.

INTRODUCTION

During the first three decades of this century a quite number of analytical solutions of tidal differential equations were given for ocean and seas with special geometrical shapes. All solutions were obtained by series expansion of a suitable functional system (Goldsbrough 1913 & 1914) or by solving an infinite system of equations (Proudman 1916 & 1931; Doodson and Proudman 1936 & 1938).

Numerical integration of tidal equations were done by Defant (1919) and Sterneck (1913) to compute tidal distribution in canals like seas, as the Red and the Adriatic Seas. It was the first time to find an agreement between computations and observations. This method was restricted only dimensional problems. Later on, the so called boundary values method has been applied to determine the tides in arbitrary two dimensional areas (Hansen, 1952).

Recently, solving Laplace's tidal equations (either analytically or numerically) for ocean basins of different geometrical shapes and depths has served in demonstrating the type of motions. In addition, it helped in investigating the sensitivity of the ocean tides with respect to some parameters such as basin width, depth, coastal configuration, Coriolis force or friction (Brettschneider, 1968; Freeman and Murty, 1976).

The present work is dealing with the investigation of the semi-diurnal tide in the Mediterranean as closed rectangular basins with different shapes and the influence of shape, and friction on the tidal patterns.

THE EQUATIONS OF MOTION AND THE NUMERICAL MODEL:

The numerical model of Hansen type (Soliman et al, 1991) has been used in the present investigation. The linear vertically integrated hydrodynamical differential equations are taken as follows:

$$\frac{\partial u}{\partial t} - fv + ku + g\frac{\partial \zeta}{\partial x} + \frac{\partial \Omega}{\partial x} = 0 \quad ..... \quad (1)$$

$$\frac{\partial v}{\partial t} + fu + kv + g\frac{\partial \zeta}{\partial y} + \frac{\partial \Omega}{\partial y} = 0 \quad ..... \quad (2$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0 \quad ..... \quad (3)$$

where:

- x.y : cartesian co-ordinates in the east and north direction respectively,
- t : time,
- ζ : water elevation of the sea surface,
- u,v : components of the depth mean current in x & y directions respectively,
- H : total depth of water,
- f : Coriolis parameter,
- k : coefficient of bottom friction in the linear form,
- g : acceleration of the earth's gravity,
- Ω : is the M2-tidal potential when the moon's declination is zero (Defant, 1960 & Pugh, 1987)

The Earth's curvature has been ignored, and the friction was supposed to be linear (ku, kv). By transforming the above equations into finite difference equations a numerical solution of the problem could be obtained if initial and boundary conditions were prescribed. The grid spacings used for the investigation were X = 90.6 km along the west - east direction and Y = 110 km along the north south direction. At the boundaries, zero normal velocities were imposed. Zero values for the water elevation and for the components of the current velocity were prescribed as the initial conditions.

RESULTS AND DISCUSSION

The basin has been suggested to take one of the following shapes (Fig. 1) :

1- A single rectangular basin:

The basin has been assumed to take a shape of a rectangular closed area extending from 6°W to 36°E longitude and from 31°N to 45°N latitude. It consists of 588 grid points with grid size 1° X 1°. The basin was proposed to have a constant depth of 500 m and 1500 m respectively. The model was then applied using friction coefficient of 10^{-5} and 10^{-6} sec^{-1} for each depth.

In such rectangular basin, with water depth of 1500 m and friction coefficient 10^{-6} sec^{-1}, one contra solem amphidromy (anticlock-wise direction) was produced (Fig. 2). The co-ranges are ellipses around the amphidromic point which is nearly located at the center of the basin. The higher amplitude observed was about 15.0 cm. This type of motion may be produced by the superposition of the longitudinal oscillations and the transverse oscillations caused by the rotation of the earth. By increasing the friction coefficient to 10^{-5} sec^{-1}, similar patterns were obtained but showing reduced range values (Fig. 3). Thus , on increasing friction co-ranges decrease.

As the water depth was reduced to 500 m, three amphidromies were obtained while the co-ranges showed different pattern (Figs. 4&5). Hence, the number of nodal lines or amphidromic points produced in a rectangular basin depends obviously on its depth.

Therefore, it is concluded that the depth of the basin has a significant influence on both the ranges and phases of the M2-tide, while friction has insignificant effect but only damping the wave. This implies that friction does not only influence the wave amplitudes but its phases too.

To estimate the influence of earth's rotation on the tidal oscillations, the motion was considered on a non-rotating earth, i.e. f=o. Figures 6&7 present the results obtained which show different patterns with respect to the above mentioned cases. Hence the rotation of the earth causes the transverse motions which lead to the formation of the contra solem amphidromies. Therefore, other types of amphidromies may be developed by other reasons like depths distribution or the transverse oscillations caused by the north south component of the driving forces. Since the influence has been ignored as the basin was assumed to be of constant depth, then the transverse oscillations might be produced by the action of the potential tidal forces which showed phase differences of $\pi/2$.

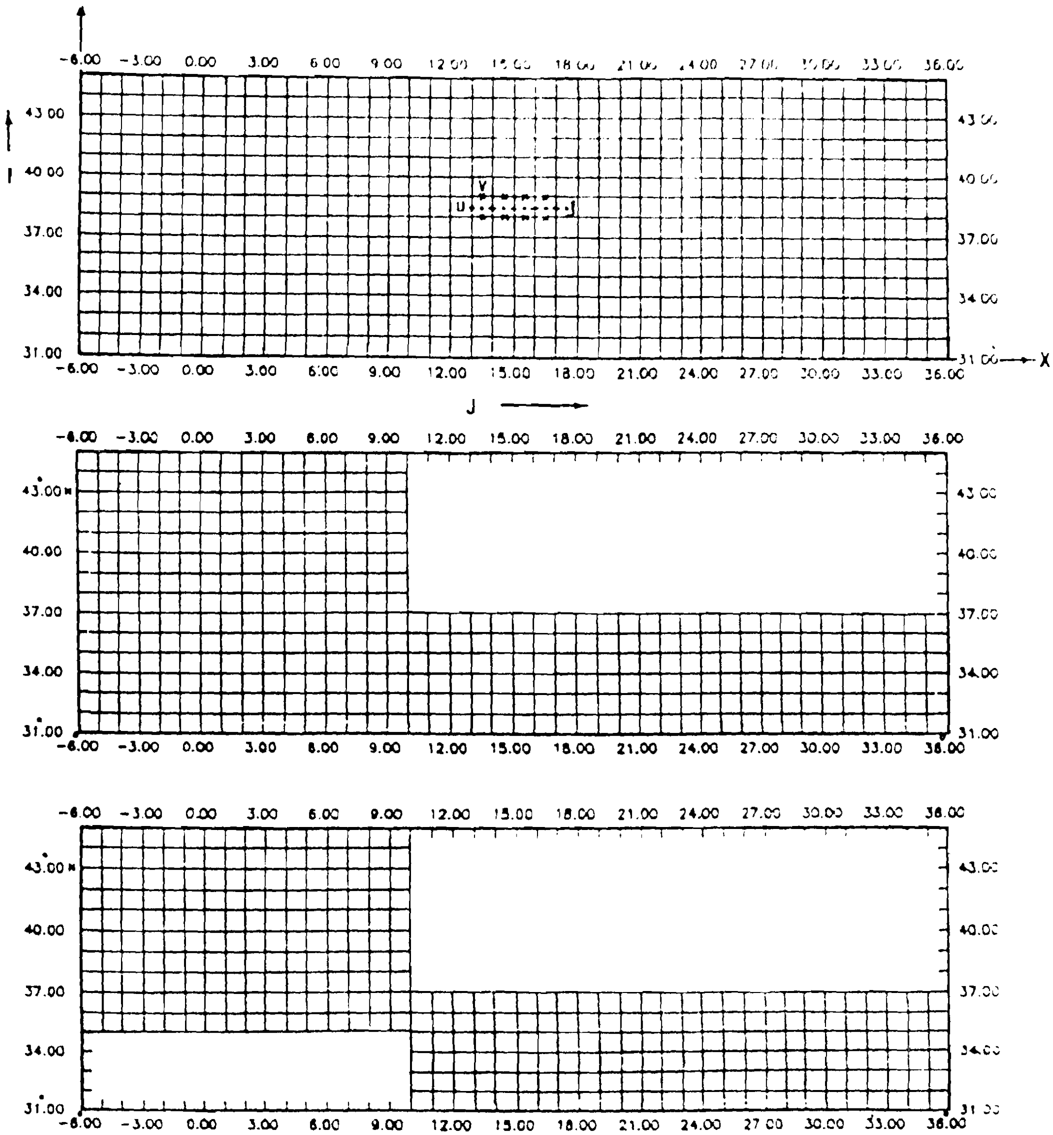
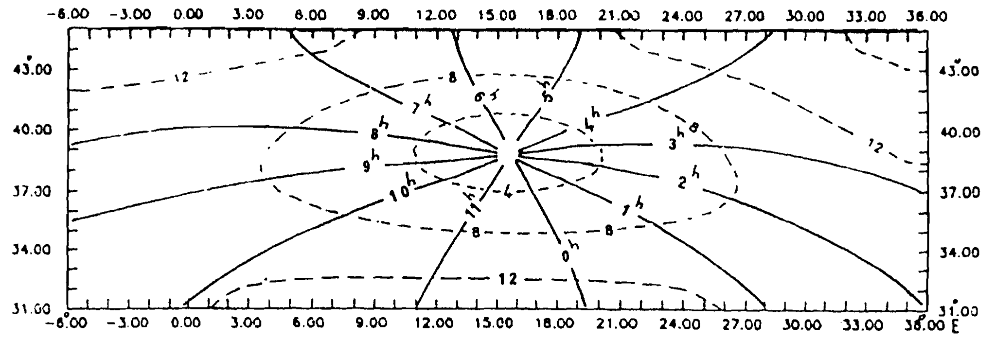


Figure 1: The different shapes of the considered rectangular basin.



igure 2: M2-Tides in a rectangular basin with constant depth of 1500m and friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

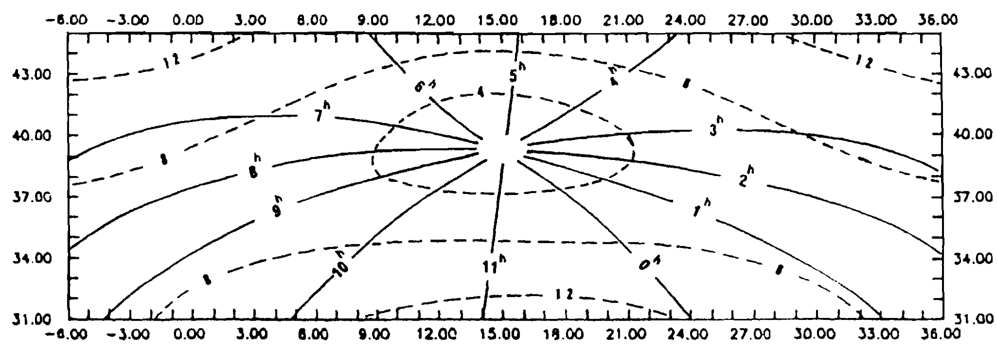


Figure 3: M2-Tides in a rectangular basin with constant depth of 1500m and friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm., and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

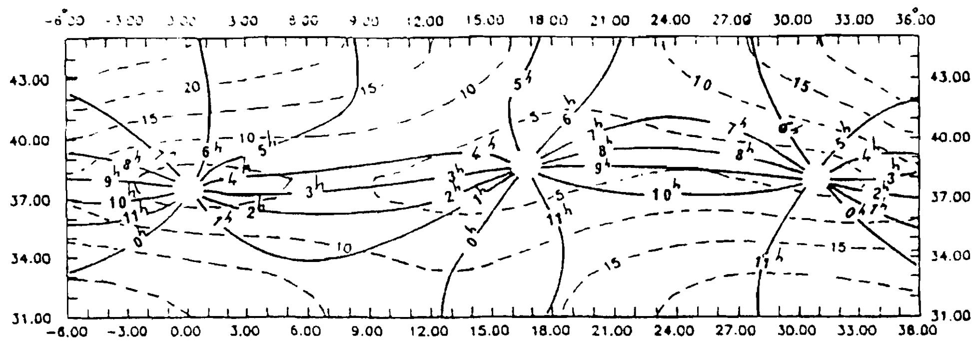


Figure 4: M2-Tides in a rectangular basin with constant depth of 500m and friction coefficient of $^{-6}$ sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

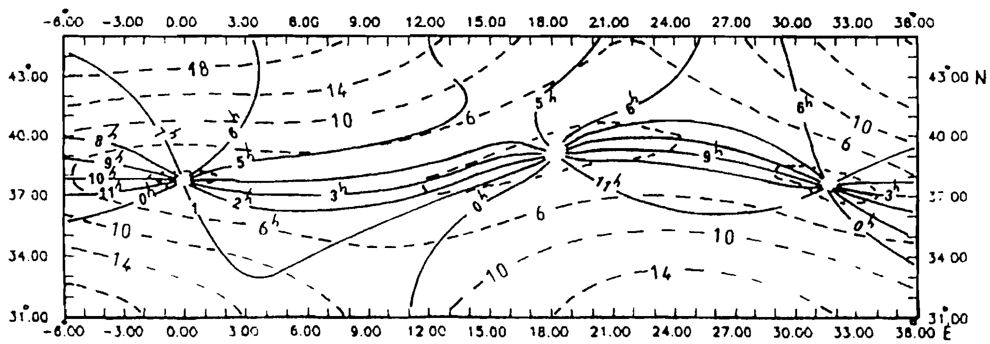


Figure 5: M2-Tides in a rectangular basin with constant depth of 500m and friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

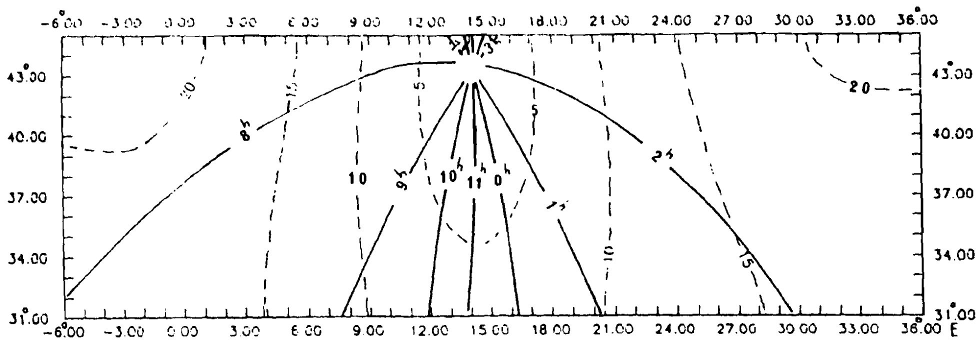


Figure 6: M2-Tides in a rectangular basin with constant depth of 1500m and friction coefficient of 10^{-6} sec^{-1} on a non-rotating earth, i.e. f=o. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

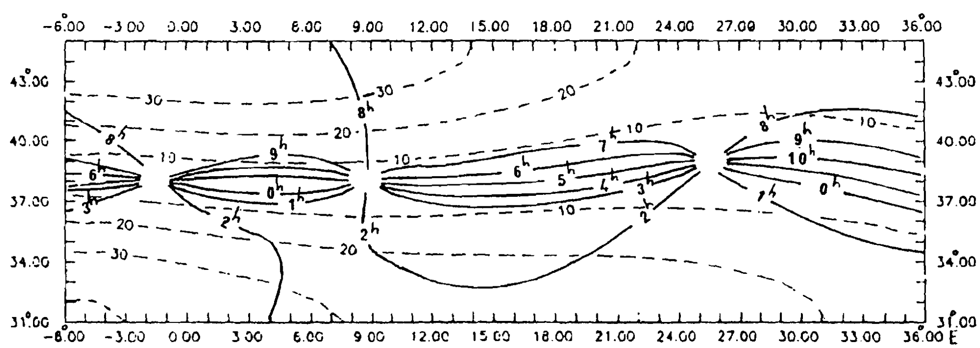


Figure 7: M2-Tides in a rectangular basin with constant depth of 500m and friction coefficient of 10^{-6} sec^{-1} on a non-rotating earth, i.e. f-o. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

2- An L- shaped basin

The Mediterranean was then assumed to take the form of L-shape. In this case, the sea was proposed to consist of two rectangular basins attached together through a wide opening of 6° width. On testing the model with a constant depth of 500 m and friction coefficient of 10^{-6} sec^{-1}, three contra solem amphidromies were produced (Fig. 8).

This pattern reveals that the amphidromies formed are either due to the transverse oscillations caused by the rotation of the earth or due to superposition of two progressive tidal waves moving along the boundaries in opposite directions.

The present pattern although it consists of three amphidromies but it differs in sense of rotation from that obtained in one rectangular basin with similar depth. The pattern of one rectangular basin indicates that the wave motion appears like a lateral standing wave, its nodal line is extending along its middle longitudinal axis with establishments of 2h in the southern part and 8h in the northern.

On increasing the depth of the L-shaped basin to 1500 m, two amphidromies were obtained instead of one in case of one rectangular basin (Fig. 9). This implies that in each basin a separate wave motion has been developed. In the western basin the contra solem amphidromy has been replaced by a cum sole amphidromy which reveals that on increasing the depth, the influences of the earth's rotation decrease or may be ignored as in the canal like areas and hence other factors were introduced to make the amphidromy rotating in the clock-wise direction, such as the potential tidal forces.

Therefore, one can conclude that the shape of the basin has a significant influence upon the tidal motion. Meanwhile, friction modifies only the co-tidal pattern and reduces the wave heights.

3- Two connected rectangular basins:

Finally, the sea was then reformed such that the new combination has resembled the real shape of the sea as far as possible. The new configuration was composed of two rectangular basins connected through an opening of 2° width. The dimensions of the western and eastern basins have almost the ratio of 3:2 and 4:1 respectively. The eastern basin in such case appeared like a canal.

The basins were assumed to be of constant depth of 500 m and then of 1500 m. Friction coefficients of 10^{-5} and 10^{-6} sec^{-1} were applied for each depth. Completely different patterns have been obtained. With constant depth 500 m and friction coefficient 10^{-6} sec^{-1}, one contra solem amphidromy was appeared in the western basin while two nodal lines were traced in the eastern one (Fig. 10). On taking the friction coefficient as 10^{-5} sec^{-1}, similar patterns were obtainedas shown in Fig. 10 but having more reasonable amplitude values particularly in the western basin (Fig. 11). As the motion was then considered

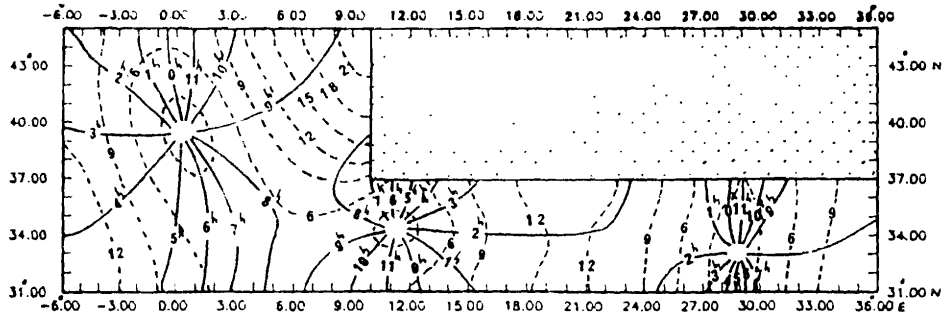


Figure 8: M2-Tides in an L -shaped basin with constant depth of 500m and friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

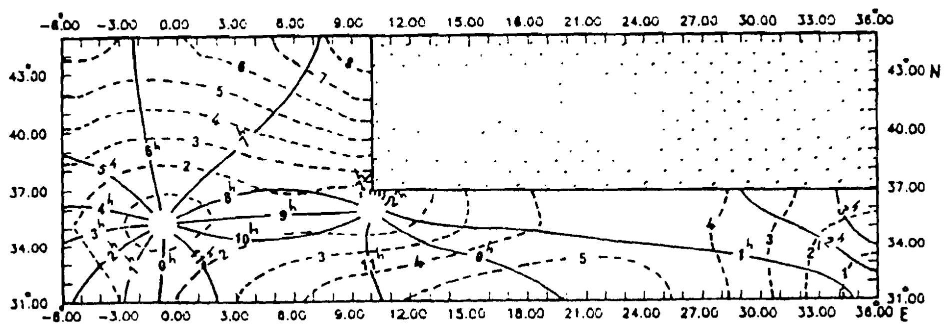


Figure 9: M2-Tides in an L -shaped basin with constant depth of 1500m and friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

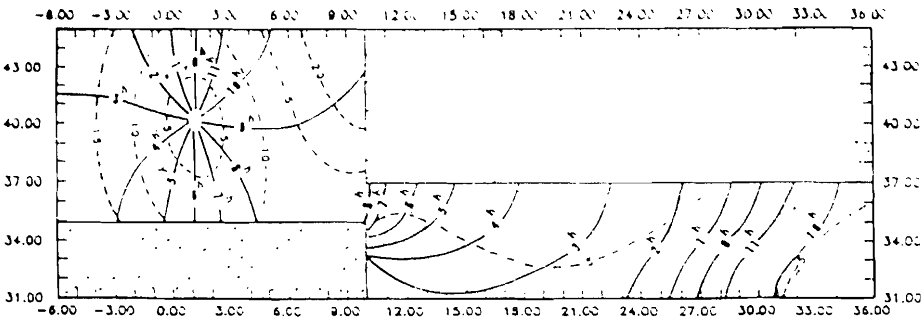


Figure 10: M2-Tides in two connected rectangular basins with constant depth of 500m and friction coefficient of 10^{-6} sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

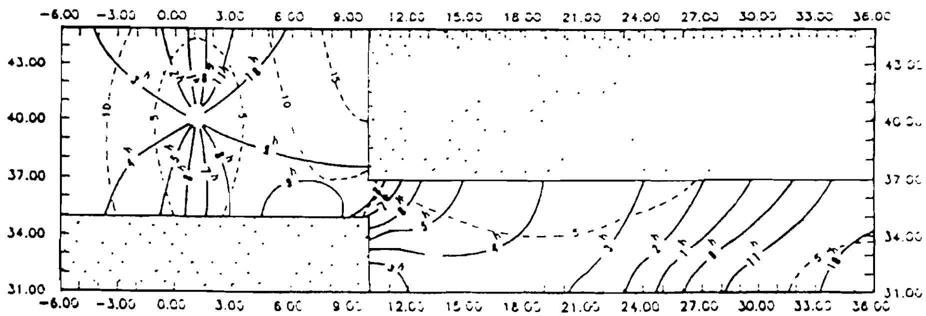


Figure 11: M2-Tides in two connected rectangular basins with constant depth of 500m and friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

on a non-rotating earth, similar pattern with f= 0 were obtained accept that instead of getting an contra solem amphidromy in the western basin a cum sole amphidromy was produced and the motion appeared as a standing wave (Figs. 12&13). Hence, the occurrence of amphidromies again does not necessarily depend on the rotation of the earth. It may also be developed by superposing longitudinal and transverse oscillations provided they have a phase difference of $\pi/2$.

With increasing the depth of the two basins to 1500 m, longitudinal oscillations have dominated either on considering the motion on rotating or non-rotating earth. At that depth, the amplitudes in the western basin were reasonable while that in the eastern basin were relatively high (Figs. 14&15).

The influence of depth as well as bottom friction on the tidal motion have been discussed by Soliman and Maiyza (1994). They mentioned that the tidal wave characteristics are independent of depth under certain circumstances. The change in amplitudes and phases with increasing depth, as mentioned in the preceding paragraph, have been explained on the basis of the free oscillations periods. These periods depend fundamentally, according to Marian's Formula ($c=\sqrt{gh}$),on the dimensions of the basin. The considerable amplitude values which computed at certain depths revealed that they are caused by the resonance of the motion induced by the driven forces with the oscillations of the basins themselves. Soliman and Maiyza concluded that at great depths, the oscillations caused by the Coriolis force are negligible and hence the produced amphidromy, if exists, is rotating in a clockwise direction (cum sole). According to this conclusion, Soliman and Maiyza found that the results obtained in the western basin for the case of two separate basins are in good agreement with the M2-tide pattern in the Black Sea obtained by Sterneck (1922).

SUMMARY AND CONCLUSIONS

A two - dimensional numerical model of Hansen type developed by Soliman et al. (1991) was used to investigate the M2-tide in rectangular closed basins as well as the influence of shape and friction on its tides. Single rectangular, L-shaped and two connected rectangular basins were considered. Linearized vertically integrated equations of motion were used. At the boundaries zero normal velocities were imposed. As initial conditions, zero water elevation values and vanishing current velocities were prescribed. The grid points were about 90.6 km and 110 km distant apart in the west - east and in the south - north directions respectively.

The investigation revealed that so long there exists a connection between two basins, co-oscillating motions will take place between them among its independent tide. The number of amphidromies formed within the sea depends on its dimension and the number of the basins involved in the sea. The transformation of the nodal lines into amphidromies depends on the type of motion as well

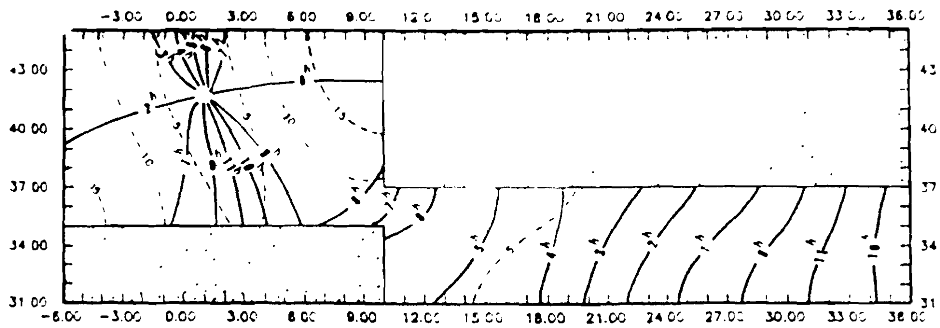


Figure 12: M2-Tides in two connected rectangular basins with constant depth of 500m and friction coefficient of 10^{-6} sec^{-1} on a non-rotating earth, i.e. f=o. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Grrenwich.

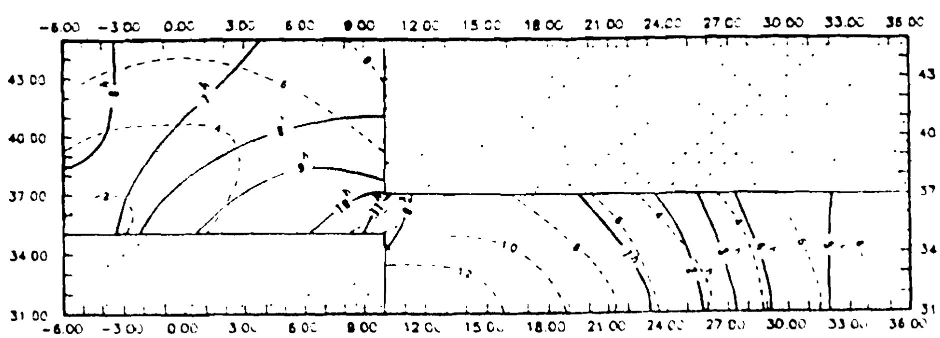


Figure 13: M2-Tides in two connected rectangular basins with constant depth of 1500m and friction coefficient of 10^{6} sec^{1}. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

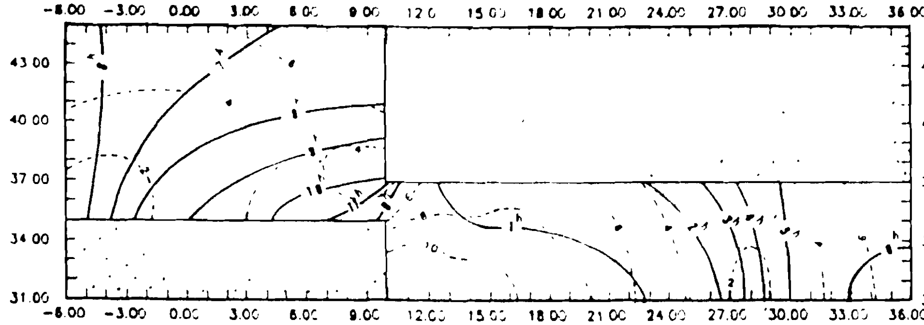


Figure 14: M2-Tides in two connected rectangular basins with constant depth of 1500m and friction coefficient of 10^{-5} sec^{-1}. The co-ranges are in cm, and the co-tidal lines-transit at Greenwich.

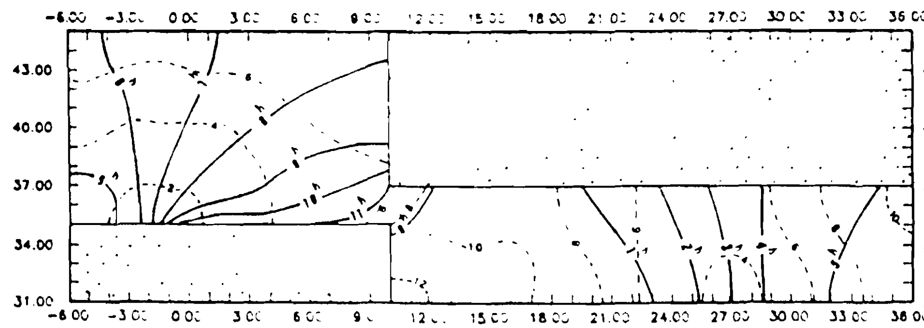


Figure 15: M2-Tides in two connected rectangular basins with constant depth of 1500m and friction coefficient of 10^{-6} sec^{-1} on a non-rotating earth, i.e. f=o. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.

a. on the transverse oscillations caused by the rotation of the earth or the external forces. The direction of rotation depends on the type of the effective forces. The main conclusions obtained are as follows :

1 The shape of the basin is significantly modifying the wave pattern in the basin. The amplitudes may be considerably increased, such effect has been examined by Soliman and Maiyza (1994).

2- By changing the depth of the basin, the phase velocity will change and hence producing other wave characteristics. The amplitudes may be considerably increased, such effect has been examined by Soliman and Maiyza (1994).

3- Neglecting the earth's rotation, the amplitudes generally increased and the amphidromic points showed changes in their locations. The wider is the basin, the larger is the difference in the water level between its longitudinal boundaries. Generally, the tidal range along the southern boundary is larger than that along the northern boundary of the basin.

4- Friction has insignificant influence not only on the phases but also on the amplitudes.

ACKNOWLEDGMENT

This work has been carried out with the support of USAID, through a contract between NJMSC and NIOF, in the framework of "STUDY OF THE CIRCULATION OF THE LEVANTINE BASIN" project. Thanks to Prof. H.K. Badawi the president of NIOF and the Principal Investigator of the project.

Thanks are also to Mr. S. Gheith for his help in writing the computer programs and following up the computation processes, and Miss S. El-Mamoney for preparing the manuscript.

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