# NUMERICAL INVESTIGATION OP CO-OSCILLATING M2-TIDE IN THE MEDITERRANEAN WITH GRID SIZE $1^{\circ} \times 1^{\circ}$. 

By

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#### Abstract

A two dimensional numerical model of Hansen type has been used to investıgate the M2-tides in the Mediterranean Sea. The direct tidal forces as well as the co-oscillating tides with the Atlantic Ocean through Gibraltar Strait were taken into account. The influence of bottom topography, coastal boundaries and friction have been considered. The results showed relatively good agreement with the observations at several localities. Deviations of the computed amplitudes and phases from observations at few localities are manly due to the large mesh size ( $1^{\circ} \times 1^{\circ}$ ) of the grid system used which did not take into account the detailed coastal configuration and depth distribution.


## INTRODUCTION

The Mediterranean Sea consists essentially of two basins with an average depth of 1750 m separated by the Sicilian Channel and the Strait of Messinia. It is connected with the Atlantic Ocean through the relatively narrow and shallow Strait of Gibraltar. Since Gibraltar Strait is so restricted, the influence of direct tidal forces, within the Mediterranean particularly in the eastern basin, may be of comparable importance to the external forces. The tides in the Mediterranean are generally comparatively weak, but although considerable tidal ranges are found in the area of Shkirya Minor in Libya (about 200 cm at Gabes and Shkirya). The most important tidal constituent in the Mediterranean is the principal lunar M2 - tide. Over most part of the sea, semidiurnal and mixed tides are observed. Diurnal tides are found only in the central region of the Adriatic Sea and in the western basin (Defant, 1961).

Based on the coastal observations of the tidal elevations. Sterneck (1913-1915) suggested that the tides in the Mediterranean could be represented by two standing waves, one in each basin with a nodal line at its center Noda! lines were also found in the Strait of Messinia and Tunis He added that the rotation of the earth may cause strong transverse oscillation. which transtorm the nodal lines into amphidromes contra solem. The co idal lines in the Adriatic Sea indicate the formation of a well developed amphidromy which may result from the superposition of longitudinal oscillations with a nodal line at Punta Bianche $\left(43^{\circ} 47^{\prime} N\right.$ Lat. and $14^{\circ} 20^{\circ}$ E Long.) and the transverse oscillations caused by the rotation of the earth (Defant. 1961). In the Aegean Sea, Sterneck suggested the existence of a nodal line somewhat north of Rhodes.

EQUATIONS OP MOTION, NUMERICAL MODEL AND BOUNDARY CONDITIONS
In the present investigation, the model used was based on the following linear vertically integrated hydrodynamical differentıal equatıons

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\begin{align*}
& \delta \mathrm{t} \quad \delta \mathrm{z} \\
& \delta v \quad \delta \boldsymbol{j} \\
& \cdots+f u+k v+g \cdots-\cdots=0  \tag{2}\\
& \delta t \quad \delta y \\
& \delta \int \delta(\mathrm{Hu}) \delta(\mathrm{HV})=0  \tag{3}\\
& \delta t \quad \delta \mathrm{z} \quad \delta \mathrm{y}
\end{align*}
$$

where:
$\mathbb{X}, \mathrm{y}$ :cartesian co-ordinates in the east and north direction respectively,
$t$ : time,
$\xi$ : water elevation of the sea surface.
$u, v$ :components of the depth mean current in $x \& y$ directions respectively
H : total depth of water.
f : Corıolis parameter,
$k$ :coefficient of bottom friction in the linear form.
g : acceleration of the earth's gravity.
$X, Y$ : the components of the tide producing torce in east and north directions respectively.

Pigure 1: The grid system used in the numerical computation with the

A finite-difference scheme, with a forward difference in time and central difference in space, has been applied (Soliman et al., 1991). A grid system with $1^{\circ} \times 1^{\circ}$ mesh size was used (F1g. 1). The potential tidal forces were taken into consideration, while the observed M2-tıdal constituent was specified at the Strait of Gibraltar. To investagate the independent M2-tide in the sea, the closed strait boundary condition of zero normal velocity was imposed. As initial conditions, the zero values for the water elevation and the components of current velocity were prescribed. A linear form of bottom friction was employed with a friction coefficient of $10^{6} \mathrm{sec}^{1}$.

## RESULTS AND DISCUSSION

In preceding works the model has been examined to investigate the M2-tide in the Mediterranean Sea as a closed basin (Soliman et al., 1991 and Eid et al., 1993). Actually, the Mediterranean is opened at its western side into the Atlantic Ocean through a very narrow and shallow strait (Gibraltar Strait). Therefore, long waves from the ocean penetrate the Strait of Gibraltar and propagate into the sea basins, such that they may influence the tides of the Mediterranean itself.

Many ideas have been demonstrated about the tides of the Mediterranean. Some investigators assumed the sea as a closed basin and hence its tides is of independent type. Some others denied the possibility of the occurrence of such independent tides. They attempted to explain the phenomena by assuming an Atlantic wave penetrating through the Strait of Gibraltar and propagating in the Mediterranean. In the present study, the direct effect of the tide generating forces as well as the oscillations penetrating into the sea from the Atlantic Ocean have been taken together to represent the driving forces of the tidal motion in the Mediterranean. To examine the influence of the bottom topography, the model has been applied on the sea with a constant depth as well as with real depths. The influence of Sicillian Strait width has also been examined.

## 1- The sea with constant depth:

Constant depths of $500,1500 \& 2000 \mathrm{~m}$ have been suggested to represent the average depths of the sea. Although its mean depth was found as 1750 m approximately, the constant depth of 500 m was proposed to examine the effect of the relatively shallow water areas on the tidal motion. Moreover, the independent m 2 -tide developed in the sea under the action of the tidal generating forces only has been estimated.

Pig. 2 presents the co-tidal and co-range lines of the M2-independent tide as obtained in the sea with a constant depth 500 m and friction coefficient $10^{6} \sec ^{1}$. Three contra solem amphidromies were found, one to the south of Balearic Islands with establishment of 5 h in the Alboran Sea, one around Sicillian Island with establishments of $10 h$ in the western basin and 4 h in the lonian Sea, and the third one as a degenerated contra-solem amphidromy around the Libian coast with establishment of $10 h$ in the eastern basin. Moreover, an additional one as a degenerated contra solem amphidromy was observed in the Adrıatic Sea at Ortona in Italy. Amplitudes of more than 5.0 cm were found in the western part of Alboran Sea, northern part of Adriatic, Aegean Sea and along the south-eastern coast of the Levantine Sea. The co-ranges pattern showed some peculiarities, if compared with the patterns obtained by Solıman and Maıyra (1993) to investıgate the influence of depth on M2-tide in the sea as assumed to be composed of two connected rectangular basins with a constant depth of 400 m and 500 m respectively. Slight changes from 400 m to 500 m caused quite clear variation in the co-tidal lines with a phase lag of about 3 h . The co-tidal pattern obtained in the eastern part of the two connected rectangular basins with constant depth of 400 m was found to be more or less similar to that mentioned in the present pattern (Fig. 2). Meanwhile, quite differences were found between the co-ranges in the western basin. These deviations may be attributed either to the influence of free oscillation in each basin or to co-oscillation with the other basins. Therefore the coastal configuration may influence the tide in the basin.

With great depths ( $1500 \mathrm{~m} \& 200 \mathrm{~m}$ ), the tidal patterns in the sea are nearly similar to that obtained in the case of one rectangular basin with constant depth of 1500 m given by Eid et al. (1993), where only one contra solem amphidromy was observed nearly at the middle of the sea. Generally, the oscillation appears as a standing wave with establishments of 7.5 h in the western basin and about ih in the eastern (Figs. 3\&4).

The Mediterranean was then allowed to communicate with the atlantic Ocean through Gibraltar Strait while the tide generating forces were still acting on the sea. A periodical oscillation corresponds to the Atlantic was maintained at the mouth of the sea according to the following relation:

$$
\begin{equation*}
J=A \cos (\sigma t+\phi) \tag{4}
\end{equation*}
$$

where:

A : is the amplitude of the periodical oscillation at the mouth of the sea (Gibraltar).

- :1s the phase of the penetrating wave as the Moon transit the Equator at Greenwich.

Regarding the sea again as a basin with constant depth of 500 m , the results were presented in Fig. 5 where three contra solem amphidromies were detected. Two of them are of degenerated forms. The patterns are more or less similar to that given when the Mediterranean was considered as a losed basin with constant


Figure 2: M2-Independent Tide in the Mediterranean Sea with a constant depth of 500 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


F1gure 3: M2-Independent Tide in the Mediterranean Sea with a constant depth of 1500 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


Figure 4: M2-Independent Tide in the Mediterranean Sea with a constant depth of 2000 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


Figure 5: M2-Co-Oscillating Tide in the Mediterranean Sea with a constant depth of 500 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar transit at Greenwich.

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depth of 500 m (Fig. 2). Only there is a phase lag of about 2 h between the two cases as a result of the penetrating waves from the Atlantic into the western basin which causes the rise of the vater level in that basin. on account of the considerable increase of the tidal ranges in the western basin, the eastern basin showed also some increase in its ranges particularly along the boundaries. Hence it is concluded that the tides in the eastern Mediterranean are mainly due to the tidal potential forces. The co-tidal lines are developed and the co-ranges are magnified by the influence of co-oscillation with the Atlantic. It is also concluded that the eastern basin is in co-oscillation with the western basin. Such conclusions contradict Darwin's idea about the tides of the Mediterranean (Defant, 1961), who assumed that the Mediterranean is so completely closed by the Strait of Gibraltar such that only independent tides can develop, so that in each basin only forced oscillations are possible. Meanwhile, it coincides to a great extent with Defant's explanation of the tidal observations (Defant, 1961). He assumed that the western basin has, beside its independent tides, co-oscillating tides not only with the Atlantic Ocean through the Strait of Gibraltar but also with the eastern basin through the Straits of Tunis and Messinia. Hence, it is of great importance to estimate the independent tide in any basin before studying its co-oscillation with the other basins.

With increasing the depth of the sea to $1000,1500 \& 2000 \mathrm{~m}$, the contra solem amphidromy that noticed around sicilian Island in the previous case did not appear with that depths. A nodal line was detected in each basin when the depth was 1000 m (Fig. 6), while a contra solem amphidromy was found in the eastern basin for the other two depths (Figs. 7 \& 8). The co-ranges showed considerable increase particularly for 2000 m . This remarkable increase in amplitude has been explained by Soliman and Maiyza (1993) as a result of approaching from the resonance conditions.

## 2- The sea with its real depths:

To investigate the M2-co-oscillating tide in the Mediterranean with its real depths, the boundary conditions were imposed at the free opening near Gibraltar as previously mentioned in the case of constant depths. Since the tidal oscillations from the Atlantic Ocean are penetrating Gibraltar Strait into the sea, the width and the depth of the strait may affect the tidal motion in the sea. Three cases were considered by suggesting the depth at the entrance of the sea as $100 \mathrm{~m}, 200 \mathrm{~m}$ and 300 m respectivel (Figs. 9-11). Friction coefficient of 10 - sec ${ }^{1}$ was assumed. Slight differences could hardly be observed between the three cases except in Ligurian Sea and the area between Balearic Islands and Corsica Island where considerable ranges were observed. This indicates that the variation of depth at the sea entrance affects mostly the amplitudes in the western basin.

The patterns obtained in the eastern part of the Alboran Sea and the region of Balearic Islands with variable depths are different from that produced with constant depth of 500 m , but is consonant to that of great constant depths. Actually, these regions are recognized with depths of more than 1000 m and hence the deep water pattern is more convenient In addition, as the eastern basin is


Figure 6: M2-Co-Oscillating Tide in the Mediterranean Sea with a constant depth of 1000 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hous of high tide after lunar-transit at Greenuich.


Pigure 7: M2-Co-Oscillating Tide in the Mediterranean Soa with a constant depth of 1500 m . The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greewich.


Figure 8: M2-Co-Oscillating Fide in the Mediterranean Sea with a constant depth of 2000 m . The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greewich.


Pigure 9: M2-Co-Oscillating fide in the Mediterranean Sea with its real depths and an entrance depth of 100 m at Gibraltar. The co-ranges are in cm, and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


Pigure 10: M2-Co-Oscillating Tide in the Mediterranean Sea with its real depths and an entrance depth of 200m at Gibraltar. The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


Pigure 11: M2-Co-0scillating Tide in the Mediterranean Sea with its real depths and an entrance depth of 300 m at Gibraltar. The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after Iunar-transit at Greenwich.
mostly characterized by its bottom topography, its tidal pattern with real depths is more relevant to that obtained with constant depth of 1000 m . Meanwhile, in the region located between the western and eastern basins, i.e. the area around sicili Island which occupied by the Straits of Tunis and Messina, the patterns of the real sea are more closer to that obtained with constant depth of 500 m .

Generally, the influence of the shallow coastal area did not appear in the present cases as a result of averaging the depths over each grid whose size was taken as $1^{0} \times 1^{0}$. The impact of bottom topography on the tidal pattern along the coast may become more recognizable when highly resolution grid sizes are performed. Therefore, it is concluded that the bottom topography has a substantial influence on the tidal patterns.

To examine the influence of friction, the model has been applied again taking the friction coefficient as $5 * 10^{-6} \mathrm{sec}^{-1}$. The patterns showed only a decrease in the ranges (Fig. 12), i.e. friction reduces only the wave heights and has insignificant influences on the wave patterns. As the width of Tunis Strait was diminished to compose only one grid, relatively high co-range values were observed at some regions in the eastern basin (Fig. 13).

Generally, the results of co-oscillating motions obtained by applying the model on the Mediterranean with its real depths are in good agreement with the observations (Table 1). The model failed in the narrow areas like Adriatic Sea, Alboran Sea and generally in the straits. This problem could be overcome on using grid system with high resolution which is the goal of the next work.

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Pigure 12: M2-Co-Oscillating tide in the Mediterranean with its real depths and friction coefficient of $5^{*} 10^{-6} \mathrm{sec}^{-1}$. The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.


Figure 13: M2-Co-Oscillating tide in the Mediterranean with its real depths and reducing the width of Tunis Strait to only one grid. The co-ranges are in cm , and the co-tidal lines correspond to lunar hours of high tide after lunar-transit at Greenwich.
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Tabie 1：cont．

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