A STATISTICAL STUDY OF LENGTH-WEIGHT RELATIONSHIP OF EIGHT EGYPTIAN FISHES

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ABSTRACT

This study is presented to help to fill a gap of a few published length/weight statistical analyses. Fish weights (W) and their logarithms (log W) gave evidences to approximate the normal distribution. One fish from eight gave an evidence of a homogeneous log W-variance. Polynomials of the third and fifth, second and third, and second and fourth degrees fitted to fish length (L) and W, log L and log W, and to L and W/L respectively previded log W-estimates significantly different from empirical values. The equation, $W = aL^n$, fitted to longer or shorter length ranges (May be stanzas), gave insignificant differences between calculated and empirical estimates and took the cubic form in two fishes. This implies that fish volume may be directly proportional to the product of L and a mean-cross-sectional area determined by two axes related to L by a power equation.

INTRODUCTION

The derivation of formulae to express length/weight relationship in fishes attracted the attention of early workers. It is chiefly used to convert length into weight or vice versa. Thus, after estimating the past length growth from scales, it is possible to estimate the past weight growth by the knowledge of length/weight relationship. Again by the knowledge of the total weight of fish landed, its length distribution, age-length key and length/weight relationship it is possible to estimate the total number of fish landed, as well as the number of fish belonging to the different age groups landed which are used for the estimation of total mortality rates and their separation into their components which are of vital importance for fishery management.

The study of the condition factor, W/L^3, or relative condition factor, $W/\hat{W}$, as indications of certain biological phenomena such as maturation or fatness came into use.

The importance of length/weight relationship was revealed, again, in the development of von Bertalanffy growth equation as well as the yield equation of Beverton and Holt (1957) from theoretical as well as from practical points of view, i.e., for directing the fisheries towards the optimum fishing. Thus, the differential form of the von Bertalanffy equation ($dw/dt = HS - kw$) was easily integrated by assuming a constant specific gravity and a cubic length/weight relationship, i.e., by assuming that $S = pL^2$ and $W = cL^3$, then the differential equation becomes, $dL/dt = E - KL$ (Beverton and Holt, 1957). This cubic length/weight relationship was used in spite of some objections (Hile, 1936 ; Frost, 1945 ; Martin, 1949 ; Le Gren, 1951).

In spite of the importance of length/weight relationship, relatively few statistical treatment have been published and the present study helps to fill this gap. Thus, the data of length and weight of eight fish species are studied hoping that this sample may be large enough to cover the different conditions. The goodness of fit of the conventional cubic equation ($W = a L^3$) and the power equation ($W = a L^n$), polynomials to L and W, to L and W/L, and to log L and log W are tested in a trial to find a mathematical expression better than the conventional equations which may show variabilities in the length/weight relationship during different ranges of length (Le Cren, 1951). Again the fit of von Bretalanffy equation to growth data of fishes having complicated length/weight relationships is studied.

MATERIALS AND METHODS

The data of eight fish, four each from the Mediterranean and the Red Seas, were collected during fisheries investigation projects. *Serranus alexandrinus* Cuv. and Val. and *Serranus gigas* Brünn were collected in 1962 during the investigation of the Mediterranean Sea Fisher es west of Alexandria by longlines fishing at the bottom. These two species were measured from tip of snout to end of coudal fin, to the nearest centimeter. The other six species were measured similarly, but to the centimeter below. *Mugil cephalus* Linn. and *Morone punctata* Linn. were collected in 1959–1961 during the study of the beach-seine fisheries on the Egyptian Mediterranean coast. The data for four fish from the Red Sea, viz. *Sardinella jussieu* (Lacépède), *Scomber japonicus* Houtt, *Clupea leiogaster* (C.V.) and *Rastrelliger Kanagurta* (C.) were obtained during the investigation of light and purse-seine fisheries of the Red Sea at El-Ghardaqa region in 1967–68.

The smaller fish, weighing less than about 100 g, were weighed by balances to the nearest 0.25 g. Larger fish were measured by other balances to the nearest gram. Grouping was carried out later. The fish were classified according to lengths into 1-centimeter classes. The classes with less than four fishes were disregarded. Fish weights as well as log W of each length class were grouped into a number of arrays so that the standard deviation was about four times as great as the array interval (Snedecor and Cochran, 1956).

DISTRIBUTION OF FISH WEIGHTS

The study of the nature of distribution and homogeneity of variance of variates are very important before the application of statistical models necessary to to study relationships between the variates.

NATURE OF DISTRIBUTION OF FISH-WEIGHTS

The hypothesis of the normal distribution was tested by the Kolmogorov-Smirnov test (Lindgren, 1962). Experience has shown that this test may indicate false significant departures from the normal distribution if the standard deviation per class-interval ratio was less than four. *Serranus alexandrinus* weights and log W of the length classes were first tested for normality because at the first the greatest number of observations were available from this fish. Weights and log W showed nonsignificant departures from the normal distribution and it was suggested that the frequencies were not large enough to reveal which of them may be the true normal. The number of observations of smaller fish like *Sardinella jussieu*, *Scomber japonicus* and *Clupea leiogaster* were much increased so that small departures from the normal distribution might be revealed. The fequency became more than 100 in three cases and more than 50 in nine cases (Appendix I-VIII). Weights and log W failed to show significant departures from the normal distribution in all cases. It is concluded that W and log W have distributions approximating very closely the normal distribution and that (one of them, probably log W, may have a true normal distribution. Le Cren 1951) mentioned that ferquency distributions of fish weights tend to Galton-Mac Alister distribution rather than the normal and that log W approximates the normal distribution.

HETEROGENEITY OF VARIANCE

The eight fish show a definite rapidly increasing W-variance with increasing length (Appendix 1-VIII). The standard deviation appears to increase linearly with the mean fish weight, so that log W may have a constant variance. Inspection of log W-variance shows that it is much more stable than W-variance.

Statical tests of homogeneity of variance assume the normal distribution of variates. Discrepancies produced by non-normality are large when two variance estimates are compared (F-test) and become larger when more than two variance estimates are compared (Bartlett test). These discrepancies become progressively larger as the number of sample variances to be compared is increased (Davies, 1956). Bartlett test was completely disregarded because the doubtful log W-normality and the large number of variance estimates to be compared in many instances. The variance ratio, F, test was considered as a non-biased test because departures from normality were statistically insignificant. The hypothesis of homogeneity of variance among n-variance estimates was tested by the F-test. The n-variance-estimates were compared in paris by the F-test. The number of possible combinations of pairs and F-values is equal to $n!/2\ (n-2)!$ An evidence of heterogeneity of variance is taken into consideration if 5% or more of F-values were significant at 5-percent level.

Denoting the log W-variance estimate at fish length L_i by S_i^2, and by S_j^2 at fish length L_j, and the variance ratio S_i^2/S_j^2 by $F_{i/j}$, evidences of heterogeneity of variance were obtained (Table 1). All fish gave evidences of heterogeneity of variance except *Scomber japonicus*. It can be concluded that heterogeneity of log W-variance is the general rule with some exceptions. The elimination of some extreme variates may result in a homogeneous log W-variance but this was not tried here.

Through out the following study, the t-test will be used as a criterion to test the significance of the difference between log W hypothetical values and empirical values, i.e. the means of log W values. The t-test assumes the normal disbution of variates in such comparisons. In cases of non-normality, the effect of skewness is rather serious but the kurtosis does not have a large effect (Davies, 1956). As log W-distribution did not show a significant departure from the normal distribution, the t test will be considered as an unbiased test.

THE LENGTH-WEIGHT RELATIONSHIP

Assuming that a fish is growing with unchanging body form, unchanging specific gravity, K_1, and denoting fish weight by W, and fish volume by V,

$$\therefore\ W = K_1\ V$$

It may be assumed that a fish has a mean-cross-sectional area, A,

$$\therefore\ V = A\ L$$

Assuming that A is directly proportional to two axes, X_1 and X_2,

$$\therefore\ A = K_2\ X_1\ X_2 \qquad \text{and}$$

$$W = K_1\ k_2\ (X_1\ X_2) \quad L$$

APPENDIX I.—Mean, Variance and Standard Deviation of W, and Log W of *Serranus alexandrinus*

Length cm	Number of observations	$\bar{W}$	S^2_W	S_W	$\overline{\text{Log}}\ W$	$S^2_{\log W}$	$S_{\log W}$
18	7	82	48	6.93	1.9250	0.00333	0.05770
20	8	111	145	12.05	2.0500	0.00214	0.04626
21	15	123	231	15.20	2.0880	0.002665	0.05160
22	16	143	534	23.10	2.1530	0.00666	0.08160
23	14	159	322	17.94	2.1960	0.00335	0.0579
24	19	174	298	17.26	2.2408	0.00307	0.05541
25	29	199	395	19.88	2.2990	0.001897	0.043555
26	28	226	804	28.35	2.3482	0.00287	0.05357
27	28	242	619	24.88	2.3803	0.00229	0.0478
28	35	271	1317	36.29	2.4336	0.00346	0.05882
29	27	297	1263	35.54	2.4694	0.00256	0.05060
30	32	326	645	25.40	2.5125	0.001290	0.03592
31	28	364	1106	33.26	2.5536	0.00230	0.04797
32	25	393	1688	41.09	2.5950	0.00250	0.0500
33	24	442	3134	55.98	2.6417	0.00254	0.05039
34	21	456	1438	37.92	2.6631	0.00148	0.0384
35	28	516	5567	74.61	2.7036	0.00508	0.07127
36	30	602	6917	83.35	2.7700	0.00351	0.05925
37	19	632	3692	60.76	2.8013	0.00177	0.04208
38	28	679	5993	77.41	2.8250	0.00241	0.04909
39	14	780	11937	109.26	2.8857	0.00392	0.06261
40	43	793	7086	84.18	2.8983	0.00230	0.04796
41	30	908	5827	76.33	2.9567	0.00164	0.04050
42	26	982	14233	119.30	2.9885	0.00291	0.05394
43	23	983	7323	85.57	2.9880	0.00187	0.04324
44	32	1077	8553	92.48	3.0280	0.00160	0.0400
45	56	1193	13179	114.80	3.0759	0.00168	0.04099
46	34	1197	18745	136.91	3.0720	0.00288	0.05367
47	39	1295	15499	124.50	3.1071	0.00217	0.0466
48	28	1386	18942	137.63	3.1375	0.00197	0.04444
49	16	1522	12365	111.20	3.1813	0.00129	0.0359
50	23	1525	23863	154.48	3.1837	0.00174	0.0417
51	10	1519	29496	171.74	3.1800	0.00303	0.0550
52	6	1821	29729	172.42	3.2667	0.00242	0.0492

$\overline{\text{Log}}\ W$: refers to the mean value of log W values.

$\bar{W}$: refers to the mean weight.

APPENDIX II.—*Serranus gigas* MEAN, VARIANCE AND STANDARD DEVIATION OF W AND LOG W

Length cm	Observation number	Mean weight	s^2_W	S_W	$\overline{\text{Log}}$ W	$S^2_{\log W}$	$S_{\log W}$
25	6	276.0	1674.4	40.92	2.43700	0.0040490	0 0636
26	5	337.6	3128.8	55.94	2.52391	0.0046990	0.06855
27	10	332.6	1385.4	37 22	2.51951	0.0023067	0.04803
28	14	361.8	1145 2	33 84	2.55647	0.0019727	0.04441
29	11	296.5	733.5	27.08	2.59733	0.0009329	0.030544
30	13	452.6	1274	35.69	2.65449	0.0011605	0.034066
31	21	509.1	2449.6	49.49	2.70496	0.0016289	0.04036
32	12	528.8	2588.5	50.88	2.72147	0.0017502	0.041835
33	18	608.4	2817.0	53.07	2.78265	0.0014481	0.03805
34	13	648.5	7043	83.92	2.80863	0.0030213	0.054966
35	18	702.8	8281.3	91.00	2.84297	0.0038127	0.061747
36	19	805.8	7400	86.02	2.90388	0.0021572	0.046446
37	8	837.3	3352	57.90	2.92193	0.0009295	0.030488
38	14	928.6	25936	161.05	2.96166	0.0059027	0.07683
39	9	1019.4	3240.3	56.92	3.00773	0.0006354	0.02521
40	15	1211	42828	206.95	3.07766	0 0047366	0.06882
41	11	1245.5	10847	104.15	3.09399	0 0012512	0.03537
42	12	1315.8	23736	154.07	3.11635	0.0028040	0.052953
43	8	1371	58555	241.98	3.12973	0.0082240	0.090678
44	6	1453	16707	129.26	3.16096	0.0014510	0.038092
45	5	1702	236970	486.80	3 21861	0.0124980	0.1118
46	5	1874	88280	297.12	3.26788	0.0056175	0.07495
47	5	1804	117230	342.39	3.24861	0.0091258	0.09553
48	4	2037.5	58091	241.02	3.30607	0.0029700	0.054498
49	5	2302	63970	252.92	3.36003	0.0022480	0.047413
50	4	2107.5	386892	622.01	3.30616	0.022680	0.1506

$\bar{\text{Log}}$ W is the mean of log W values.

APPENDIX III.– *Mugil cephalus* MEAN, VARIANCE AND STANDARD DEVIATION OF WEIGHTS (W) AND LOG W.

Length cm	Observation number	Mean weight	S^2_W	S_W	$\overline{\log W}$	$S^2_{\log W}$	$S_{\log W}$
10.5	5	11.00	0.5	0.70711	1.04067	0.000785	0.028018
12.5	4	16.75	1.5833	1.258	1.22306	0.001118	0.03343
13.5	4	22.25	4.25	2.0616	1.34595	0.001582	0.039774
16.5	4	41.25	6.25	2.500	1.61485	0.000654	0.025573
18.5	7	58.42	148.28	12.177	1.75981	0.006309	0.079429
19.5	5	73.00	182.00	13.491	1.85774	0.005902	0.076828
20.5	4	80.00	83.33	9.1286	1.90095	0.002480	0.04980
21.5	4	102.50	91.66	9.5740	2.00953	0.001592	0.039899
22.5	6	113.82	206.18	14.359	2.05343	0.002945	0.054268
23.5	8	123.12	142.43	11.934	2.08858	0.001744	0.041761
24.5	9	131.67	100.0	10.0	2.11839	0.001044	0.032311
25.5	11	157.45	290.28	17.037	2.19477	0.00231	0.048066
26.5	11	175.90	144.1	12.004	2.24432	0.000962	0.031016
27.5	7	*190.86*	*333.50*	*18.262*	*2.27896*	*0.001811*	*0.042556*
28.5	7	227.85	423.83	20.587	2.35615	0.001531	0.039128
29.5	10	241.00	337.77	18.378	2.38094	0.001010	0.031781
30.5	8	278.12	2328.11	48.250	2.43875	0.005372	0.073294
31.5	6	299.17	1384.2	37.205	2.47313	0.002886	0.05372
32.5	9	323.89	1367.37	36.977	2.50785	0.002496	0.04996
34.5	6	370.0	720.0	26.833	2.56724	0.001005	0.031701
35.5	9	411.70	3137.5	56.01	2.61106	0.003361	0.057975
36.5	5	477.0	1270	35.637	2.67754	0.001066	0.032649
38.5	10	504	687.77	26.225	2.70192	0.000484	0.0220
39.5	9	578.9	1561.1	39.51	2.76170	0.000872	0.029527
40.5	5	628.0	4120.0	64.1872	2.79611	0.002030	0.045056
43.5	5	718.0	2170.0	46.583	2.85538	0.000817	0.028583
45.5	4	797.50	7758.33	88.08	2.89982	0.002174	0.046626
47.5	7	907.9	16865.9	129.866	2.95406	0.004097	0.064008
48.5	4	955.0	966.70	31.092	2.97983	0.000195	0.013964

APPENDIX IV.– *Morone punctata* MEAN, VARIANCES AND STANDARD DEVIATION OF WEIGHTS (W) AND LOG W.

Length cm.	Observation number	Mean weight	S^2_W	S_W	$\overline{\log}$ W	$S^2_{\log W}$	$S_{\log W}$
10.5	13	10.31	1.5642	1.2507	1.00981	0.003368	0.058034
11.5	23	13.61	6.1591	2.4817	1.12711	0.006036	0.077692
12.5	23	17.35	5.055455	2.2484	1.23556	0.003439	0.058643
13.5	22	22.64	6.909047	2.6285	1.35192	0.002673	0.05170
14.5	15	26.00	7.857143	2 8031	1.41259	0.002235	0.047276
15.5	10	34.50	11.8300	3.4395	1.53591	0.001832	0.04280
16.5	6	44.33	17.06	4 1304	1.64517	0.001623	0.040286
17.5	9	49.80	46.45	6.8155	1.69323	0.00381	0.061725
19.5	4	70.00	72.667	8.5245	1.84279	0.00261	0.05110
23.5	7	135.00	1138.0	33.734	2.12487	0.005031	0.07093
24.5	4	141.25	523.0	22.869	2.14605	0.00436	0.06603
25.5	5	179.6	418.0	20.445	2.25184	0.002792	0.052859

APPENDIX V. - *Sardineula jussieu* MEAN WEIGHT, VARIANCE AND STANDARD DEVIATIONS OF WEIGHTS (W) AND LOG W.

Length cm	Observation nnmber	Mean weight	S^2_W	S_W	$\overline{\log}$ W	$S^2_{\log W}$	$S_{\log W}$
12.5	75	15.120	1.45838	1.20763	1.17817	0.0012198	0.0349257
13.5	132	19.303	2.86931	1.6939	1.28398	0.00143482	0.0378791
14.5	79	23.4684	2.765	1.6629	1.36942	0.00093601	0.030595

APPENDIX VI.—*Clupea leiogaster* MEAN, VARIANCE AND STANDARD DEVIATIONS OF WEIGHTS (W) AND LOG W.

Length cm	Observation number	Mean weight	S^2_W	S_W	$\overline{\log W}$	$S^2_{\log W}$	$S_{\log W}$
12.5	39	16.205	1.6947	1.3018	1.20829	0.00121366	0.03483
13.5	150	20.4533	2.90738	1.7051	1.30921	0.00137834	0.037126
14.5	123	24.50406	3.21967	1.79434	1.38811	0.0009768	0.031254
15.5	53	30.151	5.1346	2.2660	1.47816	0.0009883	0.031437
16.5	36	38.056	11.8857	3.4476	1.57866	0.0015859	0.039824
17.5	29	46.3103	5.7250	2.3927	1.66512	0.00050182	0.022401
18.5	17	54.9412	6.3063	2.5112	1.73946	0.0004004	0.020010

APPENDIX VII.—*Scomber japonicus,* MEAN VARIANCE AND STANDARD DEVIATION OF WEIGHTS (W) AND LOG W.

Length cm	Observation number	Mean weight	S^2_W	S_W	$\overline{\log W}$	$S^2_{\log W}$	$S_{\log W}$
15.5	8	34.0	3.71428	1.92725	1.53084	0.000644	0.025377
16.5	24	39.8333	13.270	3.6428	1.59854	0.0015352	0.0391811
17.5	61	49.7869	14.67	3.8301	1.69581	0.00116915	0.034193
18.5	66	58.3788	19.10	4.3704	1.76504	0.0010726	0.032750
19.5	65	68.4923	26.9727	5.1935	1.83443	0.0010646	0.032628
20.5	29	80.75863	41.9035	6.4733	1.90587	0.0011365	0.033713
21.5	33	92.697	57.280	7.5684	1.96565	0.0012777	0.035745
22.5	19	111.2632	55.093	7.4224	2.04543	0.0008524	0.0291957
23.5	18	124.9444	90.0552	9.4898	2.09549	0.0011432	0.0338117

APPENDIX VIII.—*Rastrelliges kanagurta* MEAN, VARIANCE AND STANDARD DEVIATION OF WEIGHTS (W) AND LOG W.

Length cm	Observation number	Mean weight	S^2_W	S_W	$\overline{\log}$ W	$S^2_{\log W}$	$S_{\log W}$
12.5	5	18.600	3.800	1.9494	1.26753	0.0021905	0.046803
13.5	34	22.3235	5.1333	2.26568	1.34667	0.0018437	0.042938
14.5	46	24.7391	2.7311	1.65261	1.39245	0.0008283	0.028781
15.5	23	30.7826	5.9045	2.42992	1.48703	0.0011567	0.03401
16.5	22	40.1818	8.5381	2.9220	1.60294	0.0009872	0.03142
17.5	35	48.6571	16.4676	4.05803	1.6856	0.00131	0.036194
18.5	26	57.1923	11.924	3.4531	1.75658	0.0006787	0.026051
19.5	45	67.000	18.4090	4.29057	1.82521	0.0007628	0.027618
20.5	9	77.777	19.450	4.41022	1.89024	0.000594	0.024372
21.5	2	97.500	60.50	7.7782	1.98831	0.001203	0.034684
22.5	11	117.1818	51.360	7.1666	2.06810	0.000733	0.027076
23.5	17	133.000	92.375	9.61119	2.12279	0.000983	0.031355
24.5	8	144.875	35.000	5.9161	2.16067	0.0003186	0.017849
25.5	6	170.667	18.26	4.2732	2.23203	0.0001186	0.01089
26.5	7	194.7142	150.90	12.2841	2.28866	0.0007505	0.027395
27.5	4	224.50	169.667	13.0251	2.35065	0.000664	0.025768

TABLE 1.—EVIDENCES OF HETEROGENEITY OF LOG W-VARIANCE

Species	No. of variance estimates	No. of possible F-values	$F_{i/j}$	No. of F-values significant at 5% level among Fi/j	Total No. of F-significant values	No. of F-significant values as evidence of heterogeneity
Serranus alexandrinus	34	561	$F_{22/j}$ $F_{i/30}$	23 16	>39	28
Serranus gigas	26	325	$F_{43/j}$ $F_{i/31}$	12 8	>20	17
Mugil cephalus	29	406	$F_{19\cdot5/j}$ $F_{30\cdot5/j}$ $F_{i/38\cdot5}$	9 9 12	>30	21
Morone punctata	12	66	$F_{11\cdot5/j}$	3	3	3
Sardinella jussieu	3	3	$F_{13\cdot5/14\cdot5}$	1	1	—
Clupea leiogaster	7	21	$F_{13\cdot5/j}$	3	>3	1
Scomber japonicus	9	36	—	—	—	3
Rastrelliger kanagurta	15	105	$F_{13\cdot5/j}$	6	>6	5

NOTE : $F_{i/j} = S^2_i / S^2_j$ where S^2_i and S^2_j refere to log W variance at L_i and L_j fish length respectively.

Assuming that the axes, X_1 and X_2, are related by a certain function to fish length, the following is a list of possible relations :

$$X_1 = b_1 L \qquad X_2 = b_2 L \quad \ldots\ldots \quad 1$$

$$X_1 = b_1 L^{n1} \qquad X_2 = b_2 L^{n2} \quad \ldots\ldots \quad 2$$

$$X_1 = a_1 + b_1 L \qquad X_2 = a_2 + b_2 L \quad \ldots\ldots \quad 3$$

$$X_1 = a_1 + b_1 L + c_1 L^2 \qquad X_2 = a_2 + b_2 L + c_2 L^2 \quad \ldots \quad 4$$

The consequencies of such possible relations are that fish weights may be related to fish lengths by the following functions respectaively :

$$W = k_1 k_2 b_1 b_2 L^3 = a L^3 \quad \ldots\ldots \quad 5$$

$$W = k_1 k_2 b_1 b_2 L^{1+n_1+n_2} = a L^n \quad \ldots\ldots \quad 6$$

$$W = a L + b L^2 + c L^3 \quad \ldots\ldots \quad 7$$

$$W = a L + b L^2 + c L^3 + d L^4 + e L^5 \quad \ldots\ldots \quad 8$$

Equation 5 is the well known cubic equation with the assumptions that the proportions of the body are constants relative to fish length. Equation 6, the power equation, is an application of Huxley's (1924) allometric formula $y = b x^k$, to describe the relative growth of various parts of the body. Equation 7 and 8 suggest fitting polynomials of the second and fourth degrees, respectively to L and W/L. Beverton and Holt (1957) suggested that a general polynomial could give a better representation than the power equation. Polynomials fitted to L and W, and log L and log W were also tried in addition to the above mentioned equations in a trial to find the best mathematical expression of length/weight relationship in fishes.

Heterogeneity of log W-variance made it more convenient to use mean weights or means of log W of the length classes as empirical estimates to fit the theoretical equations. Comparisons between theoretical and empirical estimates were carried by t-tests and an aggregate test from the available number of t-tests as shown by Fisher (1958) ; and Snedecor and Cochran (1956). The aggregate test is based on the product of the probabilities individually observed to obtain a single test (Chi-square test) of significance of deviations between the hypothetical and empirical values.

The Cubic Equation

The cubic equation is transformed to the linear form by taking logarithms of both weights and lengths.

$$\log W = \log a + 3 (\log L) \quad \ldots\ldots \quad 9$$

An estimate of 'log a' was obtained by using the equation "$\log a = \overline{\log} W - 3 (\overline{\log} L)$" where $\overline{\log}$ W is the mean of log W values and likewise for $\overline{\log}$ L. Theoretical estimates of log W were obtained by applying equation 9. *Mugil cephalus* and *Sardinella jussieu* have empirical estimates not significantly different from those derived by the cubic equation. All other fishes showed significant deviations from the cubic equation (Table 2). Therefore the cubic equation cannot be considered as a general law expressing length/weight relationship in fishes, although it is obeyed in some cases.

TABLE II.—SIGNIFICANCE OF DEVIATIONS OF THE CUBIC EQUATION ESTIMATES ($W = aL^3$) FROM EMPIRICAL ESTIMATES

Species	Lodg a-estimate	Chi-square	df	P
Serranus alexandrinus	−1.900936	$\approx$ 139.6	68	<0.001
Serranus gigas	−1.76309	$\approx$ 70.9	52	0.05−0.02
Mugil cephalus	−2.03475	$\approx$ 70.4	58	0.20−0.10
Morone punctata	−2.02980	$\approx$ 40.3	24.3	$\approx$0.02
Sardinella jussieu	−2.11142	$\approx$ 6.04	6	0.50−0.30
Clupea leiogaster	−2.07898	$\approx$ 60.6	14	<0.001
Scomber japonicus	−2.03215	$\approx$ 31.7	18	0.05−0.02
Rastrelliger kanagurta	−2.02618	> 168.0	30	<0.001

The Power Equation

The power equation, like the cubic one, is changed to the linear equation by logarithmic transformation,

$$\log W = \log a + n (\log L)$$

The parameters, log a and n, were estimated by the least squares (Table 3). Five fishes, i.e. *Serranus gigas, Mugil cephalus, Morone punctata, Sardinella jussieu* and *Scomber japonicus*, gave evidences to obey the power equation (Table 3). Three fishes, i.e. *Serranus alexandrinus, Clupea leiogaster* and *Rastrelliger kanagurta* gave hypothetical estimates according to the power equation, statistically significant from the empirical estimates (Table 3). It is clear that the power equation gives better fit than the cubic form because a greater number of species obeyed the power equation.

TABLE III.—SIGNIFICANCE OF DEVIATIONS OF THE POWER EQUATION ESTIMATES ($W = aL^n$) FROM EMPIRICAL ESTIMATES

Species	Log a-estimate	n	Chi-square	df	P
Serranus alexandrinus	−1.79257	2.92925	$\approx$ 147.9	68	<0.001
Serranus gigas	−1.89571	3.08474	$\approx$ 55.8	52	0.50−0.30
Mugil cephalus	−1.98385	2.96449	$\approx$ 67.9	58	0.20−0.1
Morone punctata	−2.25822	3.18803	$\approx$ 20.5	24	0.70−0.5
Sardinella jussieu	−2.07611	2.96874	$\approx$ 5.8	6	0.50−0.3
Clupea leiogaster	−2.24862	3.14295	$\approx$ 48.0	14	<0.001
Scomber japonicus	−2.24654	3.16668	$\approx$ 18.3	18	0.50−0.3
Rastrelliger kanagurta	−2.36740	3.26534	$\approx$ 120.8	30	<0.001

The log W-homogeneous-variance of *Scomber japonicus* is taken into consideration to carry an analysis of variance (Table 4). The deviation from the linear form of the power equation (fitted by the least squares) was found insignificant ($F = 0.0013846/0.0011357) = 1.219$, $n_1 = 7$, $n_2 = 314$, $P > 0.20$). The significance of the difference between 3.159485 and 3.0, as estimates of the slope of regression of log L on log W, was tested as shown by Graybill (1961) and found significant ($U = 0.613046/0.0013846 = 442.76$ $n_1 = 1, n_2 = 7$, λ (noncentrality) $= 221.36$, $E^2{}_{0.50} = 0.444$. The value of E^2 calculated from U, $E^2 = (n_1 U)/(n_2 + n_1 U) = 0.984$, since $0.984 > 0.4444$ therefore the probability of rejecting the hypothesis $\lambda = 0.0$, is greater than 0.997). Therefore the analysis of variance gave the same conclusion as that derived from the aggregate test from t-tests, i.e. the power equation gives a better fit than the cubic equation in the case of *Scomber japonicus*.

TABLE IVa.—Analysis Of Log W-Homogeneous Variance Of *Scomber japonicus* (Between and Within Sum Of Squares)

Variance	df	Sum of squares	Mean square
Between	8	6.239303	
Within	314	0.356616	0.0011357
Total . . .	322	6.595919	

TABLE IV b.—Significance Of Deviations From The Linear Equation "log W = 3.159485 (log L) — 2.238064" fitted by the least squares.

Variance between arrays due to	df	Sum of squares	Mean square
Linear regression	1	6.229611	
Deviation from regression	7	0.009692	0.0013846
Total . .	8	6.239303	

Note : $\sum x^2 = 0.6240628$ $\sum xy = 1.9717172$

TABLE IV c.— SIGNIFICANCE OF THE DIFFERENCE BETWEEN 3.159485 AND 3.00 AS ESTIMATES OF THE SLOPE OF REGRESSION OF LOGL ON LOG W

Veriance due to	df	Sum of squares	Mean square
Total	8	$Q = 0.622738$	
Due to B	1	$Q_2 = 0.613046$	0.613046
Error	7	$Q_1 = 0.009692$	0.0013846

The three fish with significant deviations from the power equation can obey a number of power equations applied to a number of subdivisions of length ranges studied. These subdivisions were located graphically, i.e. trial plotting, and with the aid of the aggregate test from t-tests (Table 5). Le Cren (1951) concluded that no single regression of log L on log W could adequately

TABLE V.—POWER EQUATIONS ($W = a L^n$) FITTED TO SUBDIVISIONS OF LENGTH RANGES AND SIGNIFICANCE OF DEVIATIONS BETWEEN CALCULATED AND EMPIRICAL ESTIMATES

Species	Subdivisions of langth range	Log a estimate	n	Chi-square	df	P
Serranus alexandrinus	18—30	—1.40540	2.64921	<12.2	24	>0.95
	31—42	—2.34977	3.28299	<32.3	24	>0.10
	43—45	—4.254424	4.43298	<2.20	6	>0.90
	46—52	—2.115236	3.12245	<17.6	14	>0.20
Clupea leiogaster	12.5—14.5	—2.08674	3.00	>13.8	6	<0.05
	15.5—18.5	—2.07317	3.00	>25.8	8	<0.01
	—	—	—	—	—	—
	12.5—14.5	—1.85241	2.79254	<8.25	6	>0.20
	15.5—18.5	—2.56803	3.40249	<8.02	8	>0.30
Rastrelliger kanagurta	12.5—15.5	—1.21014	2.25691	>18.4	8	<0.02
	12.5—14.5	—0.85887	1.94276	<0.64	6	>0.99
	15.5—20.5	—2.36820	3.25233	20.95	12	0.1—0.05
	22.5—27.5	—2.32631	3.24294	>22.5	12	<0.05
	22.5—24.5	—1.31785	2.50585	<5.4	6	>0.50
	25.5—27.5	—2.86111	3.62011	<4.44	6	>0.50

describe the length/weight relationship for the perch. The subdivisions of length ranges with different power equations may be explained by three reasons. 1. The probable existance of different populations with different abundance at the different lengths. 2- The existance of single populations and that subdivisions of length ranges were associated with different biological phenomeanae like maturity, different growth stanzas, or a change of specific gravity. 3- The probable existance of a combination of reasons mentioned.

Inspection of catch data and biological records has shown that the age composition of *Serranus alexandrinus* (Did not obey a single power equation) gave an evidence of a single population in the studied regions. Small and large fish of *Serranus alexandrinus* and *Serranus gigas* (Obeyed a single power eauation) were caught from all depths but the proportions of large fish increased with increasing depth in case of *Serranus alexandrinus* and with decreassing depth in case of *Serranus gigas* (Unpublished data). *Mugil cephalus varied* in length from 10.5–48.5 cm. The small fish were immature and their sex was only identified microscopically while the large fish contained ripe fish with running eggs and milt. That is *Mugil cephalus* obeyed the cubic law (A special form of the power equation) over 38 cm length range in spite of a variation of maturity with length. *Rastrelligar kanagurta* of 22.5-24.5 and 25.5-27.5 cm length ranges were collected from the same catches in December 1967 and were of the same stage of maturity, but obeyed two different power equations.

The previous discussion may show that, because of some reasons, the change of length/weight power equation with length was probably due to a change in the relation between the axes (X_1 and X_2), of the hypothetical mean-cross-sectional area, and fish length ; due to a change of specific gravity or due to both reasons. Le Cren (1951) suggested that the change of length/weight relationship of perch is correlated with maturation rather than age.

The *Clupea leiogaster* two length ranges gave a better fit to the power equation than the cubic one (Table 5), thus supporting the previous conclusion of better fits by the power equation.

Polynomials fitted to fish lengths and weights

Polynomials of the third and fifth degrees were fitted to *Serranus alexandrinus* and *Clupea leiogaster*, i.e. fish with significant deviation from a single power equation. Orthogonal polynomials were fitted to *Serranus alexandrinus* from 20 to 52 cm and to *Clupea leiogaster*, as shown by Fisher and Yates (1953). Using their notation viz.

$$W = A + B\,E_1 + C\,E_2 + D\,E_3 + E\,E_4 + F\,E_5$$

E_1, E_2 .... are polynomial functions of fish lengths and A, B. ...... are regression coefficients. The fitted polynomials were the following.

Serranus alexandrinus

$$W = 707.1515 + 50.5742\,E_1 + 0.4417\,E_2 - 0.02027\,E_3 - 0.00054\,E_4 + 0.00136\,E_5$$

Clupea leiogaster.

$$W = 32.9457 + 6.4812\,E_1 + 0.5648\,E_2 - 0.1122\,E_3 - 0.0678\,E_4 + 0.0365\,E_5$$

Serranus alexandrinus showed significant deviations between logarithms of empirical and theoretical weights calculated by the polynomial of the third degree (Chi-square > 118, df $= 66$, P < 0.001) as well as by the polynomial of the fifth degree (Chi-square > 114, dF $= 66$, P < 0.001-). *Clupea leiogaster* showed significant deviations between empirical and theoretical weights calculated by the polynomial of the third degree (Chi-square $= 26.578$, df $= 14$, P$\approx$ 0.02) and insignificant deviations by the polynomial of the fifth degree (Chi-square < 4.2, P > 0.99). The insignificant deviations are meaningless because there were seven points fitted, i.e. no degrees of freedom were left after fitting the polynomial.

It can be concluded that polynomials of the third and fifth degrees fitted to fish lengths and weights cannot be considered as general descriptions of length/weights relationship.

Polynomials fitted to L and W/L

Polynomials of the second and fourth degrees were fitted, as shown in the previous section, to L and W/L estimates of *Clupea leiogaster* and *Rastrelliger kanagurta* which showed significant deviations from a single power equation. The polynomials were the followings with the notations of the previous section.

Clupea leiogaster

$$W/L = 2.0527 + 0.2821\ E_1 + 0.0186\ E_2 - 0.0124\ E_3 - 0.0043\ E_4$$

Rastrelliger kanagurta

$$W/L = 4.118175 + 0.223348\ E_1 + 0.018402\ E_2 - 0.0001\ E_3 + 0.000188\ E_4$$

The last polynomial was fitted to data including fish of 21.5 cm length with two fish alone. The rule of distregarding mean weights obtained from less than four fish was not followed in this case, so that orthogonal polynomials could be fitted. The difference between theoretical and empirical estimates of fish weight of 21.5 cm length was insignificant (Second degree : $t = 0.351$, df $= 1$, P $>$ 0.70 ; fourth degree : $t = 0.184$, df $= 1$, P > 0.80).

Clupea leiogaster data showed significant deviations between empirical and calculated estimates by the second degree polynomial (Chi-square > 24.4, df $=$ 14, P < 0.05) and insignificant differences by the fourth degree polymnoial (Chi-square < 8.075, df $= 14$. P > 0.8). There is much doubt of the last result because there was only one degree of freedom left.

Rastrelliger kanagurta showed significant deviations from the polynomia of the second degree (Chi-square > 75, df $= 32$, P < 0.001) as well as from the fourth degree polynomial (Chi-square > 73, df $= 32$, P < 0.001).

It is concluded that polynomials of the second and fourth degrees fitted to L and W/L cannot, generally, describe length/weight relationship in fishes.

Polynomials fitted to log L and log W

It was shown previously that log L and log W data of *Clupea leiogaster* could be related by two regression lines. This lead to the hypothesis that these data may be fitted by a curved regression. Polynomials of the second and third degrees were fitted to *Clupea leiogaster* data, as shown by Fisher (1958) and were the followings:

$$\log W = 0.54436 - 1.593 (\log L) + 2.003 (\log l)^2$$

$$\log W = 0.18977 + 0.0205 (\log l) + 0.0036 (\log l)^2 + 0.750 (\log l)^3$$

The deviations between empirical and theoretical estimates of log W calculated by the second degree polynomial were significant (Chi-square > 25.7, df $= 14$, $P < 0\ 05$) and similarly with respect to estimates by the third degree polynomial (Chi-square ≈ 28, df $= 14$, $0.02 > P > 0.01$

It is concluded that polynomials of the second and third degrees fitted to log l and log W cannot, generally describe length/weight relationships in fishes.

DISCUSSION

The previous results have shown that polynomials fitted to L and W, L and W/L as well as log L and log W could not be considered as general descriptions of length/weight relationships in fishes. Two fishes from eight, viz. ***Mugil*** *cephalus* from 10.5 to 48.5 cm and *Sardinella jussieu* from 12.5 to 14.5 cm length, had length/weight relationships not significantly different from the cubic equation, $W = aL^3$. Three fish; viz. *Serranus gigas, Morone punctata,* and *Scomber japonicus* with 25-50, 10.5-25.5 and 15.5-23.5 length ranges respectively; had relationships not significantly different from the power equation, $W = aL^n$. The last three fish; viz. *Serranus alexandrinus*, *Clupea leiogaster* and *Rastrelliger kanagurta*; had relationships described by the power equation when their length ranges were subdivided into a series of smaller length ranges. The hypothesis that length/weight relationship may be described by polynomials fitted to L and W as well as L and W/L cannot be accepted. The data showed significant deviations from these polynomials and they do not explain the power equation obeyed in five fishes. Beverton and Holt (1957) poroposed that a general polynomial fitted to L and W could give a better representation than the power equation.

Therefore, length/weight relation in fishes may be generally described by the power equation. In some cases there is a single equation over a large range of length. In other cases there is a number of different power equations over a number of shorter length ranges of a fish. These length ranges may be characterized by having relatively unchanging specific gravity and relatively unchanging relation between fish length and hypothetical two axes specifying the mean mean cross-sectional-area of the fish or unchanging shape. The hypothetical axes X_1 and X_2 may be related to fish length by the power equation $X = bL^n$. Huxley(1924)used this power equation to describe the relative growth of various parts of the body.

In some cases the power equation of length/weight relationship may take the form of a cubic one and the axes specifying the mean-cross-sectional-area may be related to fish length in this case, by constant proportions, X = b L. Le Cren (1951) concluded that the perch (*Perca fluviatilis*) from time of hatching till they were 3.0 cm long has a power n, singificantly greater than that obtained from any other group. He concluded that there was a change of length/weight relationship correlated with maturation and that on the whole, the power, n, was greater than three. Hile (1936) and Martin (1949) showed that the power, n, usually lie between 2.5 and 4.0 and that in the vast majority of instances it is different from 3.0. The length ranges with unchanging length/weight relationship may be regarded as seperate stanzas. Martin (1949) related sharp breaks in relative growth lines for several species of fish to ossification and maturity. Hiatt (1947) showed that *Chanos chanos* of approximately 100 mm in body length undergoes a sharp break in relative growth of gut length. Rafail (1968) gave evidences of different ration/growth relationship of six groups of plaice according to six ranges of fish weights.

CONCLUSION

The previous discussion shows that fish weight is related to length by a power equation ($W = aL^n$) and that the cubic equation is sometimes verified. The von Bertalanffy equation of growth was integrated by assuming the cubic length/weight relationship. *Sardinella jussieu* having length/weight relation not significantly different from the cubic form, may have a growth pattern adequately described by von Bertalanffy equation. *Serranus alexandrinus*, deviating from the cubic or a single power length/weight relation, is expected to have a growth pattern not described by von Bertalanffy equation.

Denoting fish age in years by t and length at age t by L_t, the von Bertalanffy empirically fitted equations were the followings :

Serranus alexandrinus: $L_t = 72.5\,(1 - e^{-0.1536\,(t + 0.7298)})$

Sardinella jussieu: $L_t = 20.055\,(1 - e^{-0.36103\,(t + 1.22142)})$

Empirical and calculated yearly growth (Table 6) shows that growth of both fishes are very well described by von Bertalanffy equation. This equation may be regarded as an empirical formula since the assumption of isometric growth is not justified in case of *Serranus alexandrinus*. To this objection, the assumption of catabolism being proportional to fish weight, used in the differential form of von Bertalanffy equation, may be added (Ursin, 1967 ; Winberg, 1956, 1961). Gulland (1965) showed that the von Bertalanffy equation can be derived on an empirical basis, i.e. the relation between the rate of increase in length per-unit-time, dL / dt, is proportional to the difference between the actual length and a maximum length attained by the fish, L_∞.

$$dL/dt = k\,(L_\infty - L) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 10$$

k is the coefficient of proportionality.

TABLE VI.—EMPIRICAL AND CALCULATED GROWTH, ACCORDING TO VON BERTALANFFY EQUATION FOR *Serranus alexandrinus* AND *Sardinella jussieu*

Species / Age	Length (cm) at age in years					
	1	2	3	4	5	
Serranus alexandrinus	16.81	24.73	31.85	37.55	42.29	Empirical
	16.98	24.75	31.43	37.18	42.12	Calculated
Sardinella jussieu	11.0	14	15.5	17.0	18.0	Empirical
	11	13.79	15.64	16.96	17.88	Calculated

Therefore, growth parameters used for calculations of the yield equation (Beverton and Holt, 1957) should be estimated from weight data ($W^{1/3}$) and not from length, when it is proved that length/weight relation deviates significantly from the cubic form. In this case the fish may be regarded as being replaced by a hypothetical one with a cubic length/weight relationship and the assumption of isometric growth used in von Bertalanffy equation may be saved. Beverton of and Holt (1957) have shown the preferable use of weight growth curves obtained by direct measurement.

The probable variation of length/weight relationship with length shows clearly the great risk of extrapolations, e.g. L_∞ to W_∞ transformation, and that length/weight studies should be evidenced by statistical analysis.

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